

CSCI 567 Discussion

1 Multiple-choice Questions (30 points)

IMPORTANT: *Select ALL answers that you think are correct. You get 0.5 point for selecting each correct answer and similarly 0.5 point for not selecting each incorrect answer.*

- We will go through the 15 multiple-choice questions from the sample exam.
- For each question,
 - ~2 mins to read and think about the question.
 - Then we will walk through the solution.

(1) Which of the following on machine learning is correct?

- (A) Cross-validation is often used to tune the hyper-parameters of a machine learning algorithm.
- (B) Overfitting refers to the phenomenon when the training error is low but the test error is high.
- (C) One should prevent overfitting by using the test set during training.
- (D) Logistic regression is an algorithm for regression tasks.

(1) Which of the following on machine learning is correct?

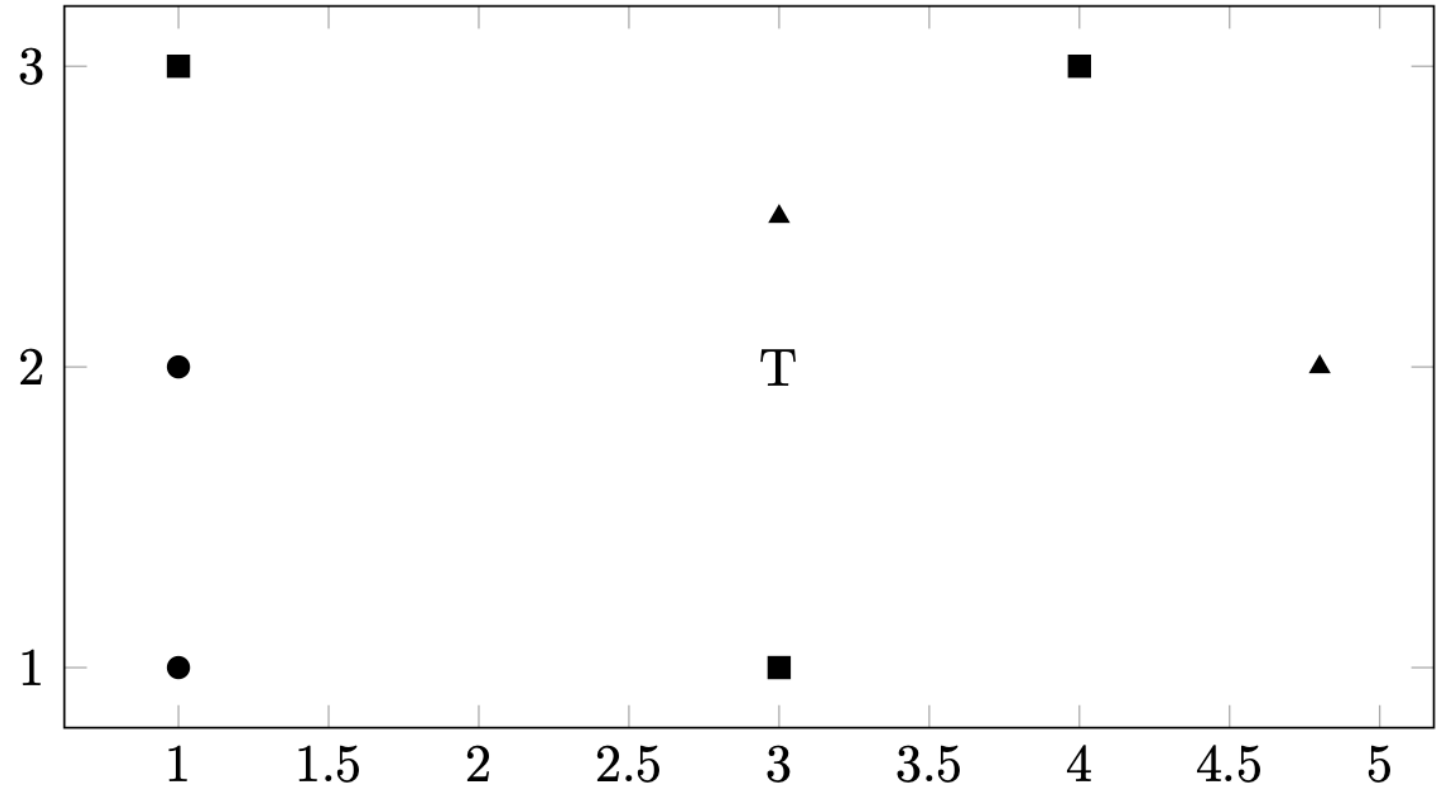
- (A) Cross-validation is often used to tune the hyper-parameters of a machine learning algorithm.
- (B) Overfitting refers to the phenomenon when the training error is low but the test error is high.
- (C) One should prevent overfitting by using the test set during training.
- (D) Logistic regression is an algorithm for regression tasks.

Answer: AB

- (C) is **incorrect**:
 - The test set should not be used in training. It is reserved for final evaluation.
 - To prevent overfitting: more training data, regularization, etc.
- (D) is **incorrect**:
 - Logistic regression is used for **classification**.

(2) Consider the following two-dimensional dataset with $N = 7$ training points of three classes (triangle, square, and circle), and additionally one test point denoted by the letter T. Which of the following configuration of the K -nearest neighbor algorithm will predict square for the test point?

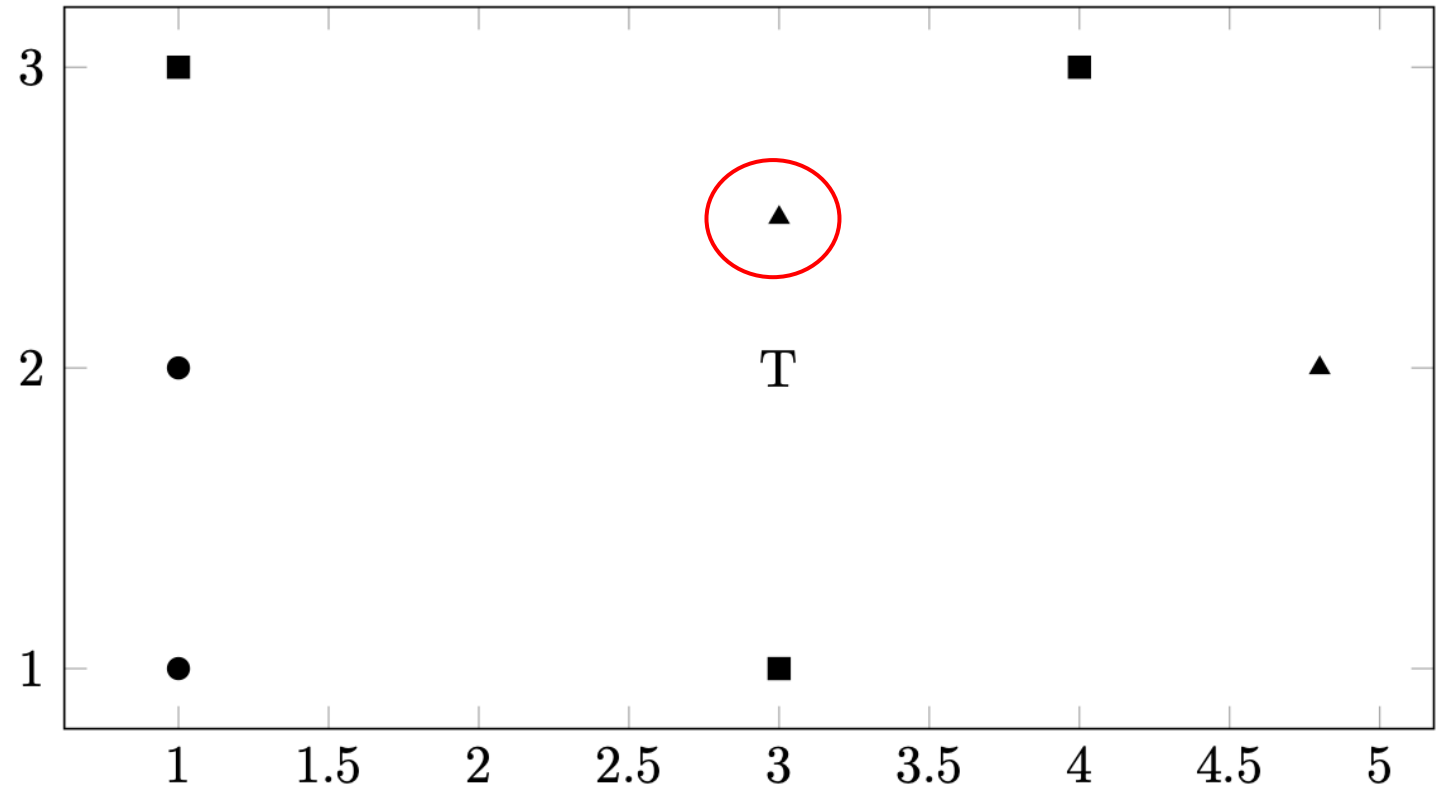
- (A) $K = 1$, L2 distance
- (B) $K = 3$, L1 distance
- (C) $K = 3$, L2 distance.
- (D) $K = 7$, any distance.



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- ~~(A) $K = 1$, L2 distance~~
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- (C) $K = 3$, L2 distance.
- (D) $K = 7$, any distance.

Answer: CD

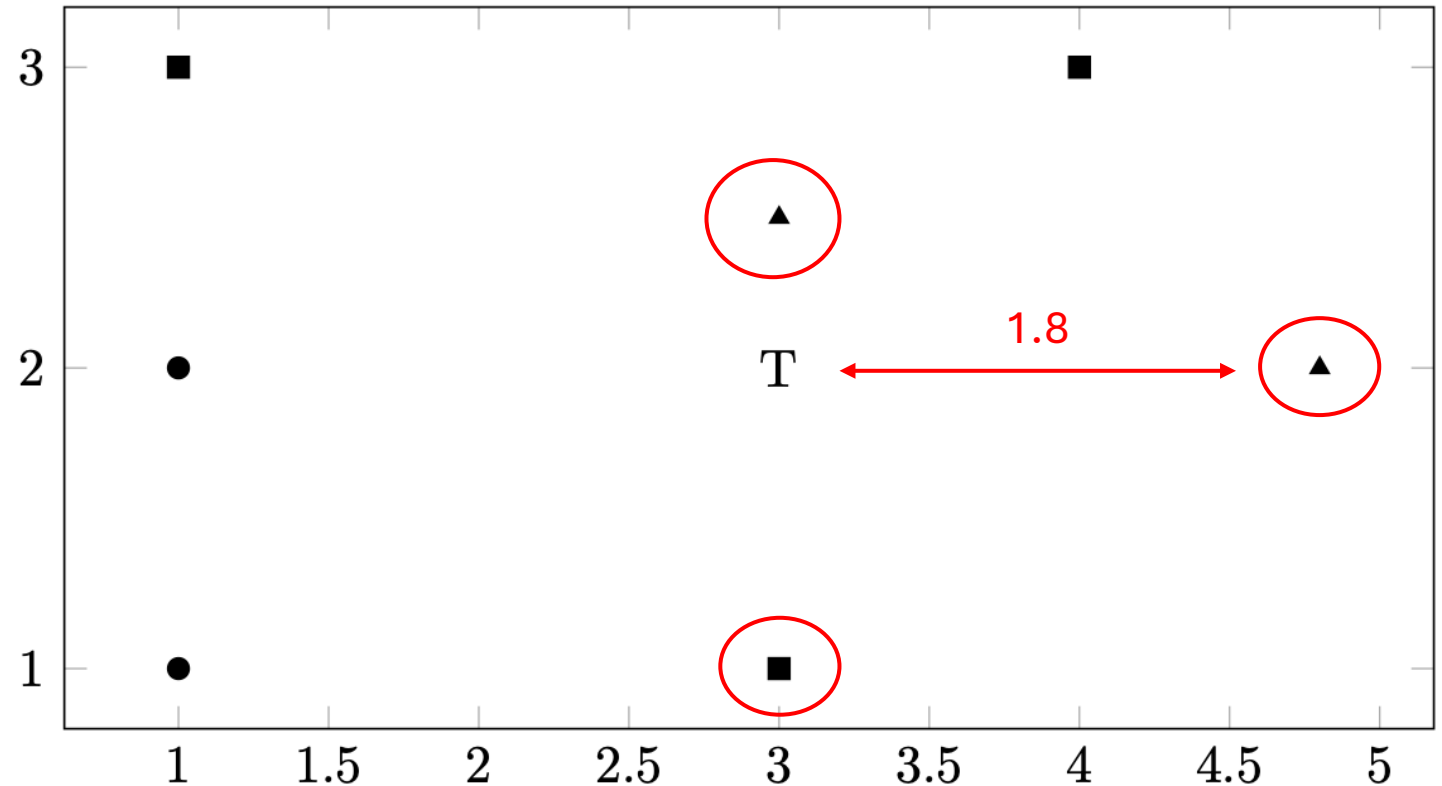


- **(A):** the single nearest neighbor is the $\blacktriangle$, so the prediction is $\blacktriangle$.

(2) Consider the following two-dimensional dataset with $N = 7$ training points of three classes (triangle, square, and circle), and additionally one test point denoted by the letter T. Which of the following configuration of the K -nearest neighbor algorithm will predict square for the test point?

- ~~(A) $K = 1$, L2 distance~~
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Answer: CD

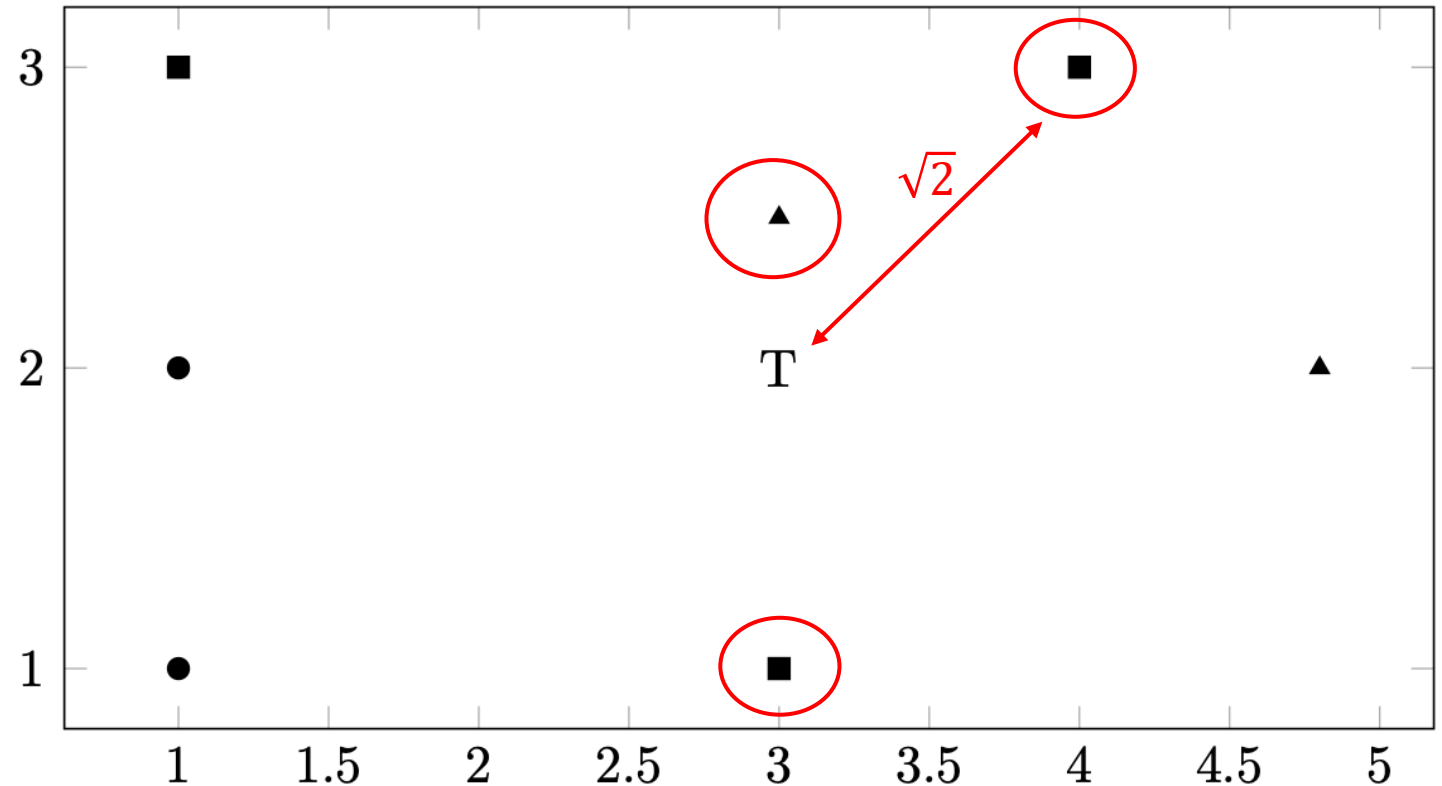


- **(B):** the 3 nearest neighbors are $\blacktriangle$, $\blacksquare$, $\blacktriangle$, the prediction is $\blacktriangle$.

(2) Consider the following two-dimensional dataset with $N = 7$ training points of three classes (triangle, square, and circle), and additionally one test point denoted by the letter T. Which of the following configuration of the K -nearest neighbor algorithm will predict square for the test point?

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Answer: CD

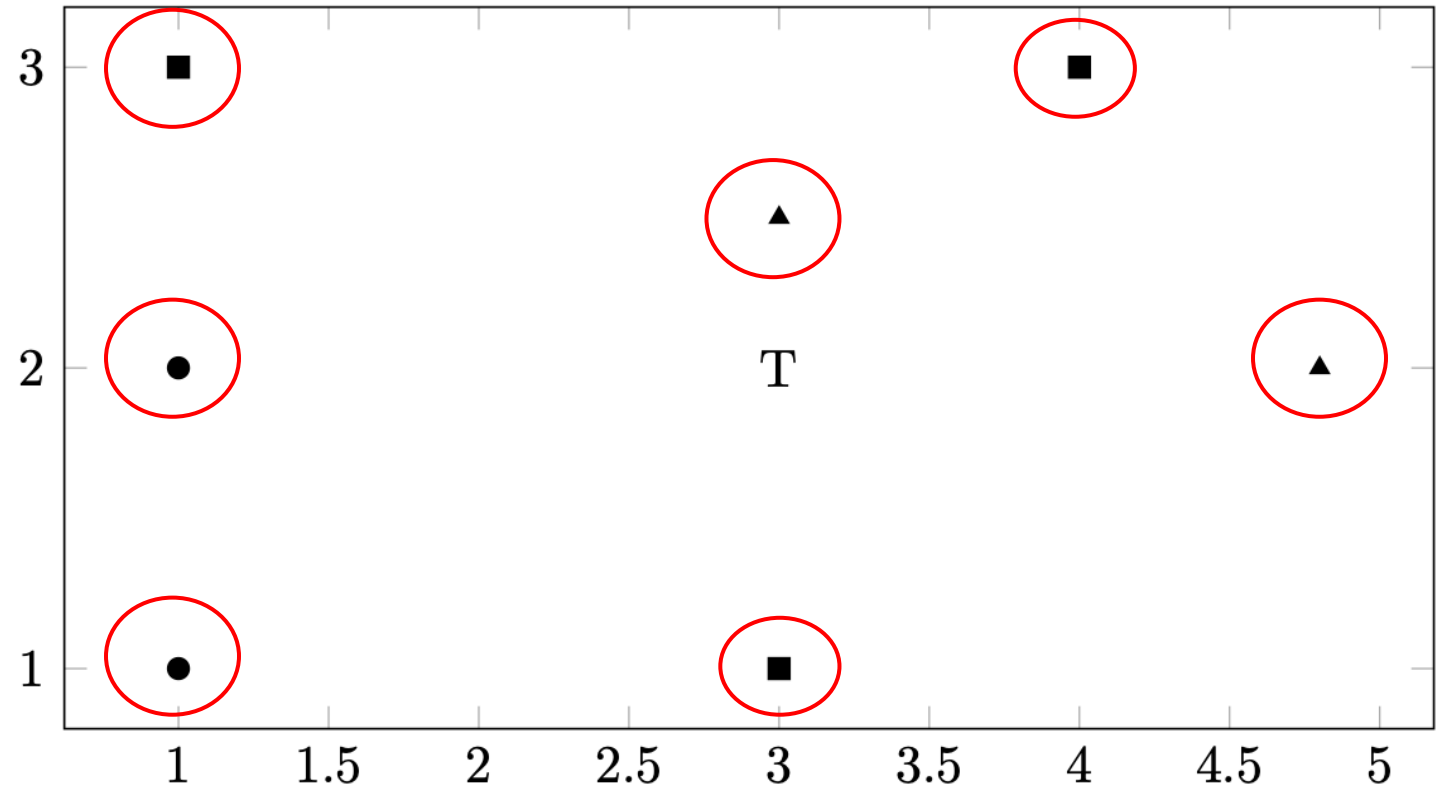


- **(C):** the 3 nearest neighbors are $\blacktriangle$, $\blacksquare$, $\blacksquare$, the prediction is $\blacksquare$.

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- ~~(A) $K = 1$, L2 distance~~
- ~~(B) $K = 3$, L1 distance~~
- (C) $K = 3$, L2 distance.
- (D) $K = 7$, any distance.

Answer: CD



- **(D):** all the points are the neighbors: ●, ●, ▲, ▲, ■, ■, ■, so the prediction is ■.

(3) Which of the following on linear regression is correct?

- (A) The least square solution has a closed-form formula, even if L1 regularization is applied.
- (B) The covariance matrix $\mathbf{X}^T \mathbf{X}$ is not invertible if the number of data points N is smaller than the dimension D .
- (C) When the covariance matrix $\mathbf{X}^T \mathbf{X}$ is not invertible, the Residual Sum of Squares (RSS) objective has infinitely many minimizers.
- (D) Linear regression is a parametric method.

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Answer: BCD

- (A) is incorrect:
 - With L2 regularization (We have worked on this before in Homework 1 Problem 2)

$$w_* = \arg \min_{w \in \mathbb{R}^D} \|Xw - y\|_2^2 + \lambda \|w\|_2^2 \quad \text{or} \quad w'_* = \arg \min_{w \in \mathbb{R}^D} \|Xw - y\|_2^2 + w^T M w \quad M = \lambda I$$

- We found its closed form by setting the gradient of $\|Xw - y\|_2^2 + w^T M w$ to 0

$$w'_* = \left(X^T X + M \right)^{-1} X^T y.$$

(3) Which of the following on linear regression is correct?

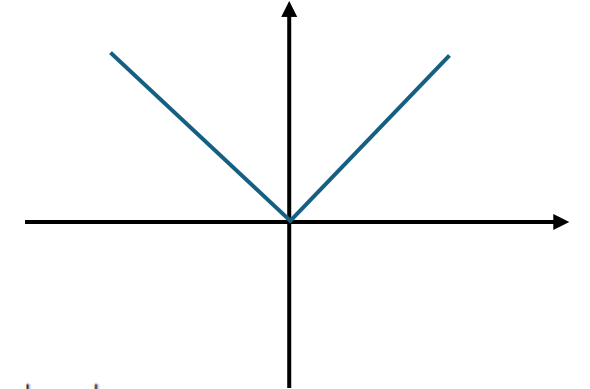
- (A) The least square solution has a closed-form formula, even if L1 regularization is applied.
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Answer: BCD

- (A) is incorrect:
 - With L1 regularization:

$$\min_w \|y - Xw\|_2^2 + \lambda \|w\|_1. \quad \|w\|_1 = \sum_i |w_i|$$

- The absolute value $|w_i|$ is not differentiable at $w_i = 0$. (L1 is not differentiable at zero)



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Answer: BCD

- (B) is correct:

What if $\tilde{\mathbf{X}}^T \tilde{\mathbf{X}}$ is not invertible

Why would that happen?

One situation: $N < D + 1$, i.e. not enough data to estimate all parameters.

- Explanation and examples can be found in 09/05 lecture slides, page 30.

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Answer: BCD

$D = 1, N = 2$

- **(C)** is correct:

sqft	sale price
1000	500K
1000	600K

Any line passing **the average** is a minimizer of RSS.

$$(\mathbf{X}^T \mathbf{X})\mathbf{w} = \mathbf{X}^T \mathbf{y}$$

$D = 2, N = 3?$

sqft	#bedroom	sale price
1000	2	500K
1500	3	700K
2000	4	800K

Again *infinitely many minimizers*.

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Answer: BCD

- (D) is correct:
 - **Parametric methods**: the size of the model does *not grow* with the size of the training set N .
 - e.g. linear regression, $D + 1$ parameters, independent of N .

$$\hat{y} = w^T x + b$$

- Parameters are $w \in \mathbb{R}^D$ and b
- No matter how many samples you get, you are only estimating $D+1$ numbers.

(4) Perceptron algorithm is applying SGD with learning rate $\eta = 1$ to the Perceptron loss, and $\mathbf{w}^* = \mathbf{0}$ is clearly a minimizer of the Perceptron loss. Which of the following statements related to these two facts is correct?

- (A) Perceptron always converges to $\mathbf{w}^* = \mathbf{0}$.
- (B) Perceptron converges to $\mathbf{w}^* = \mathbf{0}$ when it is initialized at $\mathbf{0}$.
- (C) Perceptron does not converge to $\mathbf{w}^* = \mathbf{0}$ because the learning rate does not decrease over time.
- (D) Perceptron does not converge to $\mathbf{w}^* = \mathbf{0}$ because SGD uses a stochastic gradient instead of the exact gradient.

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Answer: C

Perceptron algorithm is SGD with $\eta = 1$ applied to perceptron loss:

Repeat:

- Pick a data point x_n uniformly at random
- If $\text{sgn}(\mathbf{w}^T x_n) \neq y_n$

$$\mathbf{w} \leftarrow \mathbf{w} + y_n x_n$$

- **(A) (B)** are incorrect:
 - Even though $\mathbf{w}^* = \mathbf{0}$ is a minimizer of the perceptron loss,
 - The algorithm's update rule will move away from zero as soon as it sees data.
 - The update rule keeps adjusting and does not guarantee a return to 0.

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- **(C)** is correct:
 - The learning rate is a constant value 1 and it doesn't decrease.
 - This constant-step size SGD keeps bouncing and doesn't settle at the minimizer.

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Answer: C

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- If $\text{sgn}(\mathbf{w}^T x_n) \neq y_n$

$$\mathbf{w} \leftarrow \mathbf{w} + y_n x_n$$

- **(D)** is incorrect:
 - The primary cause is the **constant learning rate**.

(5) Which of the following statements is correct when running SGD to minimize a non-convex function?

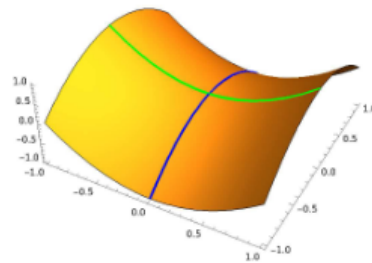
- (A) SGD with random initialization can still get stuck at saddle points.
- (B) SGD with random initialization escapes all saddle points with high probability.
- (C) As long as we run SGD long enough, it will find a local minimum.
- (D) As long as we run SGD long enough, it will find a global minimum.

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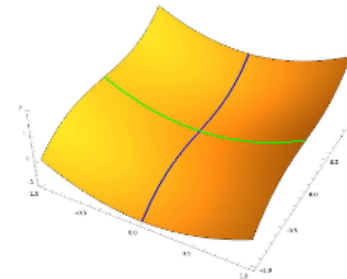
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Answer: A

- (A) is correct:
 - SGD does not guarantee an escape from all saddle points.
 - It may get stuck at flat saddle points.
 - It may hover near saddle points for a long time before it eventually escapes.
- (B) is incorrect:
 - for nonconvex objectives, can get stuck at local minimizers or “bad” saddle points (random initialization escapes “good” saddle points)



“good” saddle points



“bad” saddle points

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- (D) As long as we run SGD long enough, it will find a global minimum.

Answer: A

- **(C)** is incorrect:
 - It is possible that SGD can get stuck at some saddle points.
 - The statement is not necessarily correct.
- **(D)** is incorrect:
 - SGD converges to a stationary point that is not necessarily the global minimum.

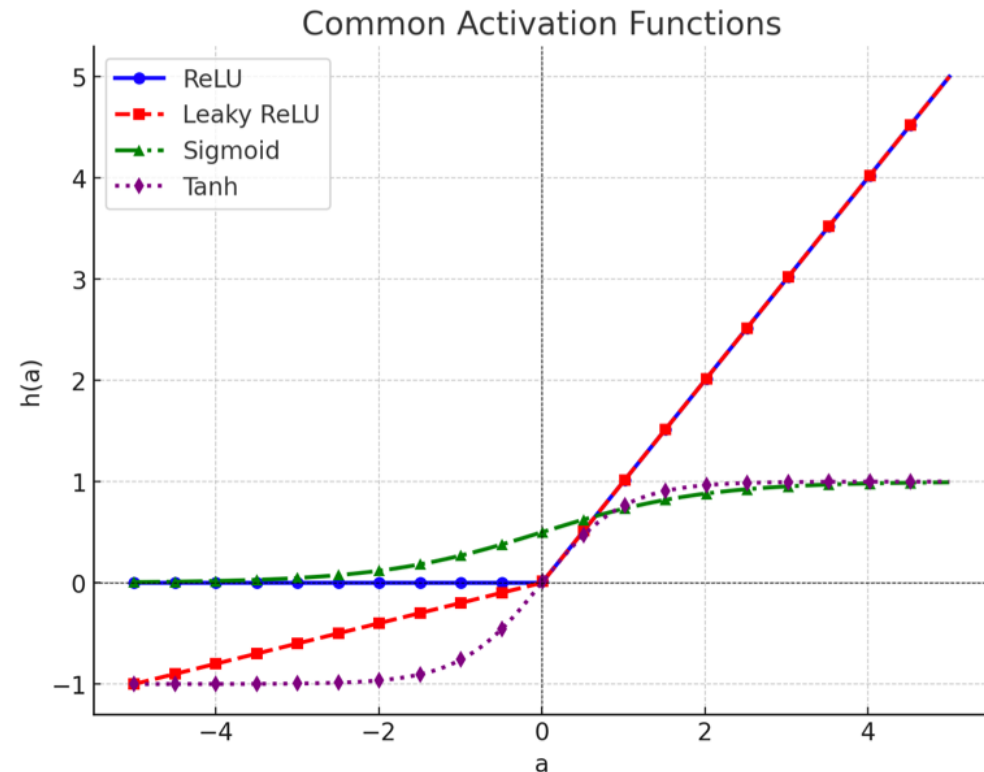
(6) Which of the following activation functions has a vanishing derivative when its input becomes too large (either too positive or too negative)? You might find the plot below useful.

(A) ReLU: $h(a) = \max\{a, 0\}$

(B) Leaky ReLU: $h(a) = \begin{cases} a & \text{if } a \geq 0 \\ 0.2a & \text{else} \end{cases}$

(C) Sigmoid: $h(a) = \frac{1}{1+e^{-a}}$

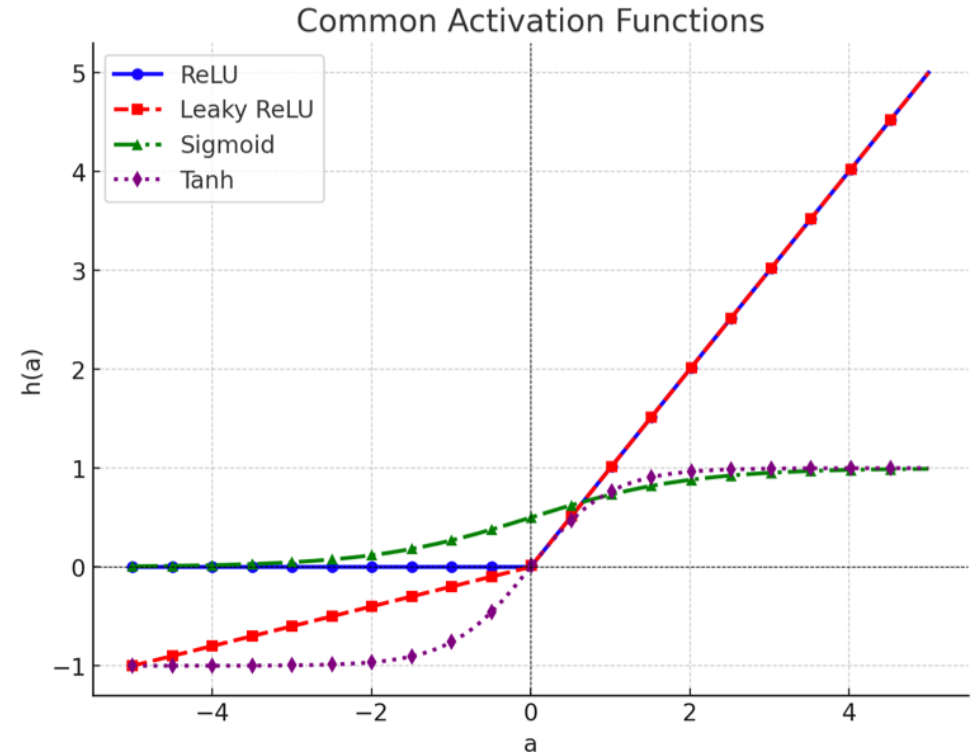
(D) Tanh: $h(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$



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- (C) Sigmoid: $h(a) = \frac{1}{1+e^{-a}}$
- (D) TanH: $h(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$

Answer: ACD



- **Vanishing gradient problem:** during training, the gradients (used to update weights) become extremely small.
- **(A):** derivative = 1 if $a > 0$; derivative = 0 if $a < 0$. For large negative inputs, derivative is 0.
- **(B):** derivative = 1 if $a > 0$; derivative = 0.2 if $a < 0$.
- **(C):** derivative is $h(a)(1 - h(a))$.
 - For very large positive inputs, $h(a) \rightarrow 1$, so derivative $\rightarrow 0$.
 - For very large negative inputs, $h(a) \rightarrow 0$, so derivative $\rightarrow 0$.
- **(D):** derivative is $1 - h(a)^2$, for very large positive (negative) inputs, $h(a)^2 \rightarrow 1$, derivative $\rightarrow 0$.

(7) Which of the following about neural nets is correct?

- (A) A fully-connected neural net with a million neurons can approximate any continuous function.
- (B) Data augmentation is useful for preventing overfitting when training a neural net.
- (C) Momentum and adaptive learning rate are useful for speeding up the training of a neural net.
- (D) One should keep running Backpropagation until the training error goes down to 0.

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Answer: BC

- (A): incorrect.

Universal approximation theorem (Cybenko, 89; Hornik, 91):

A feedforward neural net with a single hidden layer can approximate any continuous functions.

It might need a huge number of neurons though, and *depth helps!*

The theorem says you can approximate with such a neural net with certain size, but does not say there is a fixed-size that can approximate all.

Overfitting is very likely since neural nets are too powerful.

Methods to overcome overfitting:

- (B): correct.

- data augmentation
- regularization
- dropout
- early stopping
- ...

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Answer: BC

Important tricks to optimize neural nets

- **(C)**: correct.

Many important tricks on top on Backprop

- **mini-batch**: randomly sample a batch of examples to form a stochastic gradient (common batch size: 32, 64, 128, etc.)
- **batch normalization**: normalize the inputs of each neuron over the mini-batch (to zero-mean and one-variance; *c.f.* Lec 1)
- **adaptive learning rate**: scale the learning rate of each parameter based on some moving average of the magnitude of the gradients
- **momentum**: make use of previous gradients (taking inspiration from physics)

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Answer: BC

- **(D)**: incorrect.
 - Can lead to overfitting.
 - The goal is **not to** get perfect training accuracy, but to **generalize** well on unseen data.

(8) Suppose that a convolution layer takes a 4×6 image with 3 channels as input and outputs a $3 \times 4 \times 10$ volume. Which of the following is a possible configuration of this layer?

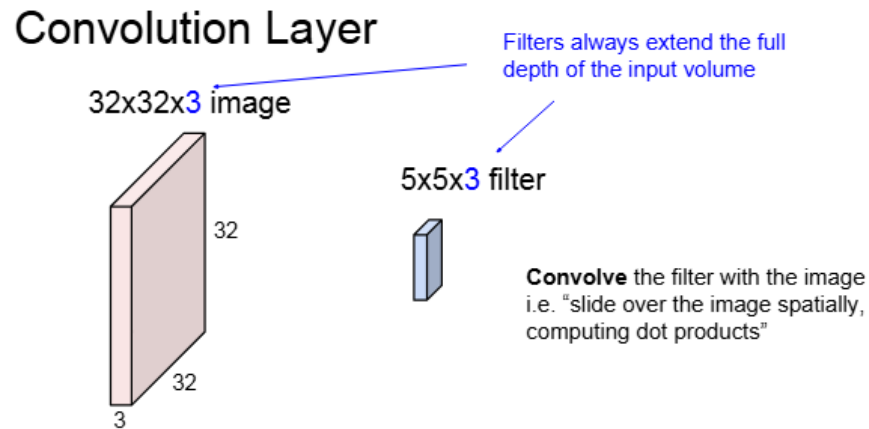
- (A) Ten 2×3 filters with depth 10, stride 1, and no zero-padding.
- (B) Ten 2×2 filters with depth 3, stride 2, and 1 pixel of zero-padding.
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- (D) One 2×3 filter with depth 10, stride 1, and no zero-padding.

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- (D) One 2×3 filter with depth 10, stride 1, and no zero-padding.

Answer: B

- **Input: $4 \times 6 \times 3$, output: $3 \times 4 \times 10$**
- **(A):** incorrect, filter depth = 10, which does not match the 3 channel input.



Input: a volume of size $W_1 \times H_1 \times D_1$

Hyperparameters:

- K filters of size $F \times F$
- stride S
- amount of zero padding P (for one side)

Output: a volume of size $W_2 \times H_2 \times D_2$ where

- $W_2 = (W_1 + 2P - F)/S + 1$
- $H_2 = (H_1 + 2P - F)/S + 1$
- $D_2 = K$

#parameters: $(F \times F \times D_1 + 1) \times K$ weights

Common setting: $F = 3, S = P = 1$

(8) Suppose that a convolution layer takes a 4×6 image with 3 channels as input and outputs a $3 \times 4 \times 10$ volume. Which of the following is a possible configuration of this layer?

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- (B) Ten 2×2 filters with depth 3, stride 2, and 1 pixel of zero-padding.
- (C) Ten 2×2 filters with depth 3, stride 2, and 2 pixels of zero-padding.
- (D) One 2×3 filter with depth 10, stride 1, and no zero-padding.

Answer: B

- **Input: $4 \times 6 \times 3$, output: $3 \times 4 \times 10$.**
- **(B):** Correct.
 - Number of filters = 10, which matches the output depth D_{out} 10.
 - Filter depth = 3, which matches the 3 channel input D_{in} .
 - $W_{out} = \frac{4+2*1-2}{2} + 1 = 3$, which matches the output W.
 - $H_{out} = \frac{6+2*1-2}{2} + 1 = 4$, which matches the output H.

Input: a volume of size $W_1 \times H_1 \times D_1$

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- amount of zero padding P (for one side)

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- $W_2 = (W_1 + 2P - F)/S + 1$
- $H_2 = (H_1 + 2P - F)/S + 1$
- $D_2 = K$

#parameters: $(F \times F \times D_1 + 1) \times K$ weights

Common setting: $F = 3, S = P = 1$

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Answer: B

- **Input: $4 \times 6 \times 3$, output: $3 \times 4 \times 10$.**
- **(C):** incorrect.
 - $W_{out} = \frac{4+2*2-2}{2} + 1 = 4$, which does not match the output W.
 - $H_{out} = \frac{6+2*2-2}{2} + 1 = 5$, which does not match the output H.
- **(D):** incorrect.
 - Number of filters = 1, which does not match the 10 output depth.

Input: a volume of size $W_1 \times H_1 \times D_1$

Hyperparameters:

- K filters of size $F \times F$
- stride S
- amount of zero padding P (for one side)

Output: a volume of size $W_2 \times H_2 \times D_2$ where

- $W_2 = (W_1 + 2P - F)/S + 1$
- $H_2 = (H_1 + 2P - F)/S + 1$
- $D_2 = K$

#parameters: $(F \times F \times D_1 + 1) \times K$ weights

Common setting: $F = 3, S = P = 1$

(9) How many parameters do we need to learn for the following network structure? An $8 \times 8 \times 3$ image input, followed by a convolution layer with 8 filters of size 3×3 (stride 1 and 1 pixel of zero-padding), then another convolution layer with 4 filters of size 2×2 (stride 2 and no zero-padding), and finally an average pooling layer with a 2×2 filter (stride 2 and no zero-padding). (Note: the depth of all filters are not explicitly spelled out, and we assume no bias/intercept terms used.)

- (A) 144 (B) 344 (C) 348 (D) 360

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(A) 144 (B) 344 (C) 348 (D) 360

Answer: B

- Convolution layer 1:
 - Parameters = $(3 * 3 * 3) * 8 = 216$.
 - There is no “+1” since we assume no bias/intercept terms used.
- Convolution layer 2:
 - Parameters = $(2 * 2 * 8) * 4 = 128$
 - Previous convolution layer output D is 8.
- Average pooling layer:
 - Parameter = 0.
- $216 + 128 + 0 = 344$.

Pooling

Similar to a filter, except

- depth is always 1
- different operations: average, L2-norm, max
- no parameters to be learned

Input: a volume of size $W_1 \times H_1 \times D_1$

Hyperparameters:

- K filters of size $F \times F$
- stride S
- amount of zero padding P (for one side)

Output: a volume of size $W_2 \times H_2 \times D_2$ where

- $W_2 = (W_1 + 2P - F)/S + 1$
- $H_2 = (H_1 + 2P - F)/S + 1$
- $D_2 = K$

#parameters: $(F \times F \times D_1 + 1) \times K$ weights

Common setting: $F = 3, S = P = 1$

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(10) What is the final output dimension of the last question?

- (A) $4 \times 4 \times 1$ (B) $4 \times 4 \times 4$ (C) $2 \times 2 \times 1$ (D) $2 \times 2 \times 4$

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(10) What is the final output dimension of the last question? **Answer: D**

(A) $4 \times 4 \times 1$ (B) $4 \times 4 \times 4$ (C) $2 \times 2 \times 1$ (D) $2 \times 2 \times 4$

• Convolution layer 1:

$$\begin{aligned} \bullet \quad W_{c1} &= \frac{8+2*1-3}{1} + 1 = 8 \\ \bullet \quad H_{c1} &= \frac{8+2*1-3}{1} + 1 = 8 \\ \bullet \quad D_{c1} &= 8 \end{aligned} \quad (8,8,8)$$

• Convolution layer 2:

$$\begin{aligned} \bullet \quad W_{c2} &= \frac{8+2*0-2}{2} + 1 = 4 \\ \bullet \quad H_{c2} &= \frac{8+2*0-2}{2} + 1 = 4 \\ \bullet \quad D_{c2} &= 4 \end{aligned} \quad (4,4,4)$$

• Average pooling layer:

$$\begin{aligned} \bullet \quad W_p &= \frac{4+2*0-2}{2} + 1 = 2, H_p = \frac{4+2*0-2}{2} + 1 = 2, D_p = 4 \\ \bullet \quad &\text{Pooling does not change the depth.} \end{aligned} \quad (2,2,4)$$

Input: a volume of size $W_1 \times H_1 \times D_1$

Hyperparameters:

- K filters of size $F \times F$
- stride S
- amount of zero padding P (for one side)

Output: a volume of size $W_2 \times H_2 \times D_2$ where

- $W_2 = (W_1 + 2P - F)/S + 1$
- $H_2 = (H_1 + 2P - F)/S + 1$
- $D_2 = K$

#parameters: $(F \times F \times D_1 + 1) \times K$ weights

Common setting: $F = 3, S = P = 1$

(11) Suppose that k_1 and k_2 are two kernel functions with $\phi_1 : \mathbb{R}^D \rightarrow \mathbb{R}^M$ and $\phi_2 : \mathbb{R}^D \rightarrow \mathbb{R}^M$ being the corresponding feature maps. Which of the following is the corresponding feature map ϕ for the product of k_1 and k_2 (which we know is also a kernel function)?

(A) $\phi(\mathbf{x}) = \phi_1(\mathbf{x})\phi_2(\mathbf{x})^\top \in \mathbb{R}^{M \times M}$

(B) $\phi(\mathbf{x}) = \phi_1(\mathbf{x})^\top \phi_2(\mathbf{x}) \in \mathbb{R}$

(C) $\phi(\mathbf{x}) = (\phi_1(\mathbf{x}), \phi_2(\mathbf{x})) \in \mathbb{R}^{2M}$

(D) $\phi(\mathbf{x}) = \phi_1(\mathbf{x}) \circ \phi_2(\mathbf{x}) \in \mathbb{R}^M$ (element-wise product)

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Answer: A

Kernel functions

Definition: a function $k : \mathbb{R}^D \times \mathbb{R}^D \rightarrow \mathbb{R}$ is called a *kernel function* if there exists a function $\phi : \mathbb{R}^D \rightarrow \mathbb{R}^M$ so that for any $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^D$,

$$k(\mathbf{x}, \mathbf{x}') = \phi(\mathbf{x})^\top \phi(\mathbf{x}')$$

Can be seen as a kind of similarity measure.

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Answer: A

$$k_1(x, x')k_2(x, x') = \phi_1(x)^\top \phi_1(x') \phi_2(x)^\top \phi_2(x').$$

By definition

$$= \phi_1(x)^\top \underbrace{(\phi_1(x')\phi_2(x')^\top)}_{M \times 1} \underbrace{\phi_2(x)}_{1 \times M}.$$

$s = \text{tr}(s)$. where s is a scalar

$$= \text{tr}(\phi_1(x)^\top \phi_1(x')\phi_2(x')^\top \phi_2(x)).$$

Now we can use $\text{tr}(AB) = \text{tr}(BA)$.

$$= \text{tr}(\phi_2(x)\phi_1(x)^\top \phi_1(x')\phi_2(x')^\top).$$

$$\langle A, B \rangle = \text{tr}(A^\top B) = \sum_{i,j} A_{ij}B_{ij}.$$

$$= \langle \phi_1(x)\phi_2(x)^\top, \phi_1(x')\phi_2(x')^\top \rangle.$$

$$\begin{aligned} k(x, x') &= \langle \phi(x), \phi(x') \rangle. \\ &= \phi(x)^\top \phi(x') \end{aligned}$$

Put A into a kernel function gives:

$$\langle \phi_1(x)\phi_2(x)^\top, \phi_1(x')\phi_2(x')^\top \rangle$$

(12) Which of the following on SVM is correct? In case you need a reminder, the primal formulation of SVM is $\min_{\mathbf{w}, b, \{\xi_n\}} C \sum_n \xi_n + \frac{1}{2} \|\mathbf{w}\|_2^2$ subject to $1 - y_n(\mathbf{w}^\top \phi(\mathbf{x}_n) + b) \leq \xi_n$ and $\xi_n \geq 0$ for all n , and the dual formulation can be found in Problem 4.1.

- (A) The larger the hyper-parameter C , the smaller the amount of L2 regularization.
- (B) The larger the hyper-parameter C , the larger the amount of L2 regularization.
- (C) To handle multiclass classification, one can use the one-versus-one reduction together with SVM.
- (D) The α coefficients obtained by SVM satisfy $\sum_{n:y_n=+1} \alpha_n = \sum_{n:y_n=-1} \alpha_n$.

$$\begin{aligned} \max_{\alpha_1, \dots, \alpha_N} \quad & \sum_{n=1}^N \alpha_n - \frac{1}{2} \sum_{m=1}^N \sum_{n=1}^N y_m y_n \alpha_m \alpha_n k(\mathbf{x}_m, \mathbf{x}_n) \\ \text{s.t.} \quad & \sum_{n=1}^N \alpha_n y_n = 0 \quad \text{and} \quad \alpha_n \geq 0, \quad \forall n \end{aligned}$$

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Answer: ACD

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- **(A):** correct.
 - A larger C means that we focus more on correctly classifying examples.
 - The optimizer would rather get weaker L2 regularization to achieve that.
- **(B):** incorrect.

(12) Which of the following on SVM is correct? In case you need a reminder, the primal formulation of SVM is $\min_{\mathbf{w}, b, \{\xi_n\}} C \sum_n \xi_n + \frac{1}{2} \|\mathbf{w}\|_2^2$ subject to $1 - y_n(\mathbf{w}^T \phi(\mathbf{x}_n) + b) \leq \xi_n$ and $\xi_n \geq 0$ for all n , and the dual formulation can be found in Problem 4.1.

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Answer: ACD

- **(C):** correct.
 - For each pair of classes, train a SVM to separate these two classes.

$$\begin{aligned} \max_{\alpha_1, \dots, \alpha_N} \quad & \sum_{n=1}^N \alpha_n - \frac{1}{2} \sum_{m=1}^N \sum_{n=1}^N y_m y_n \alpha_m \alpha_n k(\mathbf{x}_m, \mathbf{x}_n) \\ \text{s.t.} \quad & \sum_{n=1}^N \alpha_n y_n = 0 \quad \text{and} \quad \alpha_n \geq 0, \quad \forall n \end{aligned}$$

One-versus-one (OvO)

(picture credit: [link](#))

Idea: train $\binom{C}{2}$ binary classifiers to learn “**is class k or k' ?**”.

Training: for each pair (k, k') ,

- relabel examples with class k as $+1$ and examples with class k' as -1
- *discard all other examples*
- train a binary classifier $h_{(k,k')}$ using this new dataset

(12) Which of the following on SVM is correct? In case you need a reminder, the primal formulation of SVM is $\min_{\mathbf{w}, b, \{\xi_n\}} C \sum_n \xi_n + \frac{1}{2} \|\mathbf{w}\|_2^2$ subject to $1 - y_n(\mathbf{w}^T \phi(\mathbf{x}_n) + b) \leq \xi_n$ and $\xi_n \geq 0$ for all n , and the dual formulation can be found in Problem 4.1.

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Answer: ACD

$$\begin{aligned} \max_{\alpha_1, \dots, \alpha_N} \quad & \sum_{n=1}^N \alpha_n - \frac{1}{2} \sum_{m=1}^N \sum_{n=1}^N y_m y_n \alpha_m \alpha_n k(\mathbf{x}_m, \mathbf{x}_n) \\ \text{s.t.} \quad & \sum_{n=1}^N \alpha_n y_n = 0 \quad \text{and} \quad \alpha_n \geq 0, \quad \forall n \end{aligned}$$

- (D): correct.

$$\begin{aligned} \sum_{n=1}^N \alpha_n y_n &= 0 & \sum_{n=1}^N y_n \alpha_n &= \sum_{y_n=+1} (+1) \alpha_n + \sum_{y_n=-1} (-1) \alpha_n & \sum_{y_n=+1} \alpha_n &= \sum_{y_n=-1} \alpha_n \\ & & &= \sum_{y_n=+1} \alpha_n - \sum_{y_n=-1} \alpha_n = 0 \end{aligned}$$

(13) Which of the following about decision trees is correct?

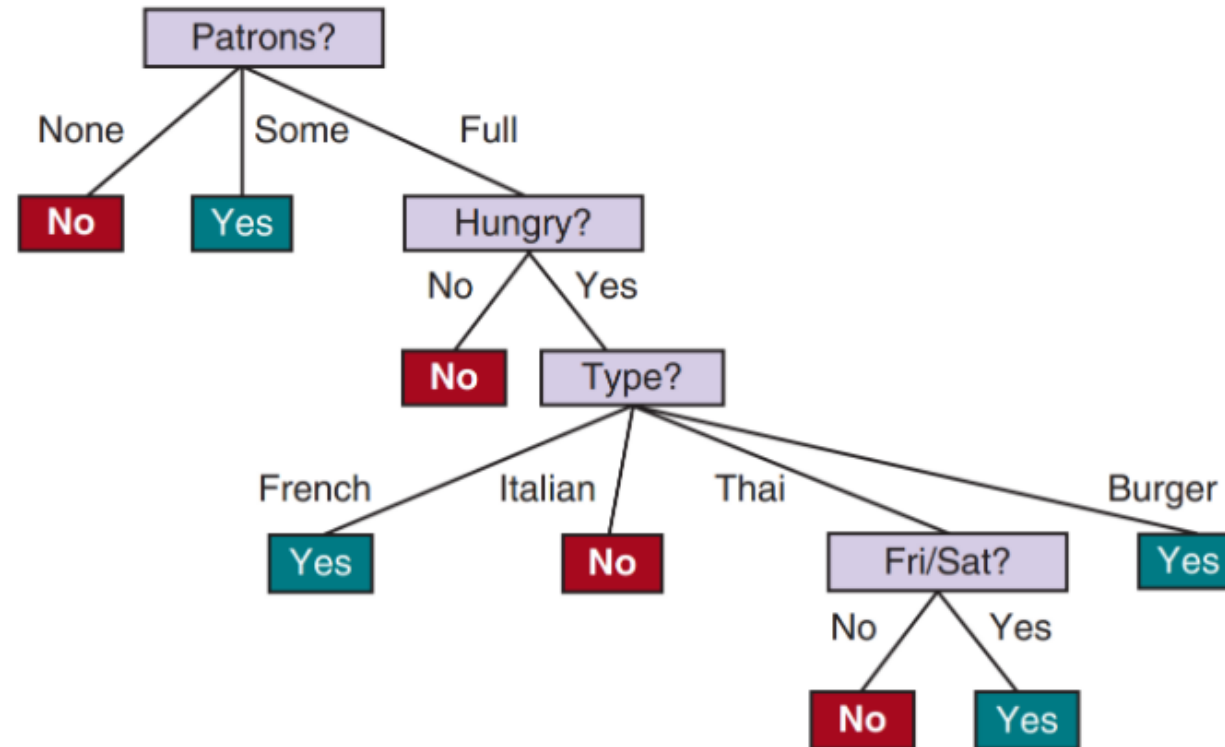
- (A) Good interpretability is a key advantage of decision trees.
- (B) Decision tree algorithms are usually implemented using recursion.
- (C) Shannon entropy can be used to measure the uncertainty of a node when building a decision tree.
- (D) Random forest is a random ensemble of decision trees.

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Answer: ABCD

- **(A):** correct.



Again, very easy to interpret.

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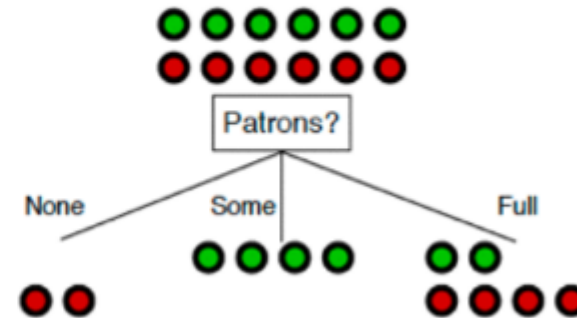
Answer: ABCD

- **(B):** correct.
 - The process of building a decision tree (e.g., ID3) involves recursively splitting nodes based on chosen features until a stopping criterion is met.

Repeat recursively

Split each child in the same way.

- but no need to split children “none” and “some”: they are pure already and become leaves
- for “full”, repeat, focusing on those 6 examples:



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Measure of uncertainty of a node

Answer: ABCD

- **(C):** correct.

It should be **a function of the distribution of classes**

- e.g. a node with 2 positive and 4 negative examples can be summarized by a distribution P with $P(Y = +1) = 1/3$ and $P(Y = -1) = 2/3$



One classic uncertainty measure of a distribution is its *(Shannon) entropy*:

$$H(P) = - \sum_{k=1}^C P(Y = k) \log P(Y = k)$$

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Answer: ABCD

- **(D):** correct.

Random forest is an **ensemble** of trees:

- each tree is built using *a bootstrapped dataset* (that is, a set of points randomly sampled from the training set with replacement)

(14) Consider a binary dataset with 50 positive examples and 50 negative examples. Decision stump $\mathcal{T}_1$ splits this dataset into two children where the left one has 20 positive examples and 40 negative examples, while another decision stump $\mathcal{T}_2$ results in a left child with 25 positive examples and 25 negative examples. Which of the following is correct? (Recall that entropy is defined as $H(P) = -\sum_{k=1}^C P(Y = k) \log P(Y = k)$.)

- (A) The entropy of the left child of $\mathcal{T}_1$ is $\frac{1}{3} \log 3 + \frac{2}{3} \log \frac{3}{2}$.
- (B) The entropy of the right child of $\mathcal{T}_1$ is $\frac{1}{4} \log 4 + \frac{3}{4} \log \frac{4}{3}$.
- (C) The entropy of either child of $\mathcal{T}_2$ is $\frac{1}{2} \log 2 + \frac{1}{2} \log 2$.
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Answer: ABCD

- **(A):** correct.
 - Left child node: 20 +, 40 -, => $p(+) = 20/60 = 1/3$, $p(-) = 40/60 = 2/3$.

$$H_L = -\left(\frac{1}{3} \log \frac{1}{3} + \frac{2}{3} \log \frac{2}{3}\right) = \frac{1}{3} \log 3 + \frac{2}{3} \log \frac{3}{2}.$$

- **(B):** correct.
 - Right child node (the rest of examples): 30 +, 10 -, => $p(+) = 3/4$, $p(-) = 1/4$.

$$H_R = -\left(\frac{3}{4} \log \frac{3}{4} + \frac{1}{4} \log \frac{1}{4}\right) = \frac{1}{4} \log 4 + \frac{3}{4} \log \frac{4}{3}.$$

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measured by the **conditional entropy**:

$$\begin{aligned}
 H(Y | A) &= \sum_a P(A = a) H(Y | A = a) \\
 &= \sum_a P(A = a) \left(- \sum_{k=1}^C P(Y | A = a) \log P(Y | A = a) \right)
 \end{aligned}$$

Pick the feature that leads to the smallest conditional entropy.

Answer: ABCD

- **(C):** correct.
 - Each child node: 25 +, 25 -, => p(+) = p(-) = 1/2.

$$H = - \left(\frac{1}{2} \log \frac{1}{2} + \frac{1}{2} \log \frac{1}{2} \right) = \frac{1}{2} \log 2 + \frac{1}{2} \log 2$$

- **(D):** correct.

- T1: $0.6 H_L + 0.4 H_R = 0.6(0.6365) + 0.4(0.5623) = 0.607.$

- T2: $0.5 \cdot H + 0.5 \cdot H = H = \log 2 = 0.693 > 0.607$

(15) Which of the following about boosting is correct?

- (A) Boosting is guaranteed to achieve zero training error.
- (B) AdaBoost is often resistant to overfitting.
- (C) AdaBoost never overfits.
- (D) The idea of boosting is to repeatedly reweight the examples so that “difficult” ones get more attention.

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Answer: BD

- (A): incorrect. It is not guaranteed.
- (B): correct.

Resistance to overfitting

However, *very often AdaBoost is resistant to overfitting*

- (C): incorrect.

Overfitting

When T is large, the model is very complicated and overfitting can happen

- (D): correct.