CSCI567 Machine Learning (Fall 2025)

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Exam 2 Logistics

Date: Friday, Dec 5th

Time: 2:00-4:00pm (plus another 20 mins for uploading)

Location: THH 201 (Initial A-R) and SGM 101 (Initial S-Z)

Individual effort, close-book (no cheat sheet), no calculators or any other electronics, *but need your phone to upload your solutions to Gradescope from 4:00-4:20pm*

Exam 2 Coverage

Coverage: mostly Lec 7-11 (just see the sample)

Seven problems in total

- one problem of 15 multiple-choice multiple-answer questions
  - *please note the new instructions!!*
- six other homework-like problems, each has a couple sub-problems
  - clustering, EM, HMM, RNN/transformer, bandits, RL

Outline

1. Review of last lecture
2. Basics of Reinforcement learning
3. Deep Q-Networks and Atari Games
4. Policy Gradient, Actor-Critic, and AlphaGo

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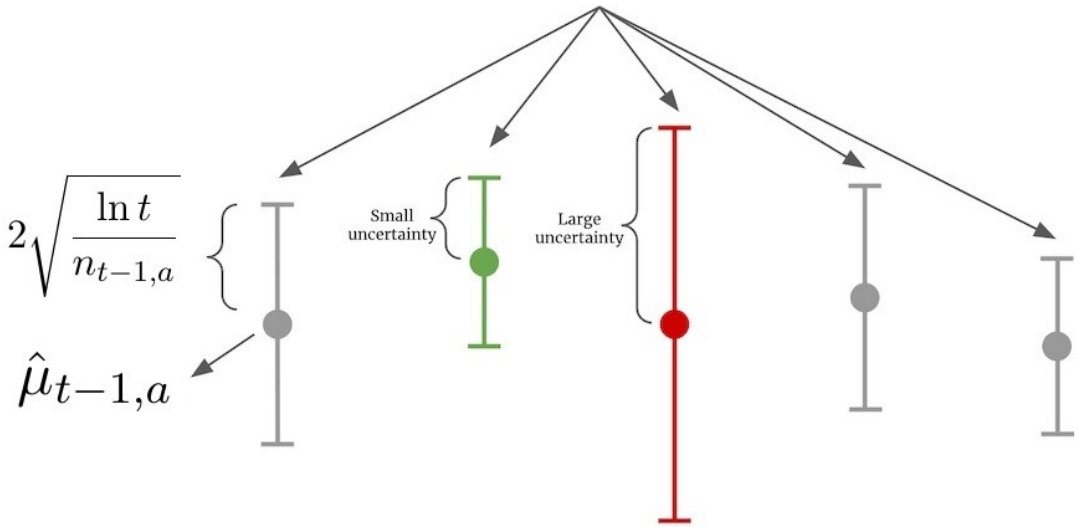
UCB for multi-armed bandits

Adaptive exploration-exploitation trade-off via optimism

Upper Confidence Bound (UCB) algorithm

For $t = 1, \ldots, T$, pick $a_t = \text{argmax}_a$ $\text{UCB}_{t,a}$ where

$$\text{UCB}_{t,a} \triangleq \hat{\mu}_{t-1,a} + 2\sqrt{\frac{\ln t}{n_{t-1,a}}}$$



Self-play for dueling bandits (preference feedback)

Exp3 for dueling bandits (selecting b_t)

Input: a learning rate parameter $\eta > 0$

For $t = 1, \ldots, T$,

- compute arm distribution $\boldsymbol{q}_t = \text{softmax}\left(-\eta \sum_{\tau=1}^{t-1} \boldsymbol{\ell}_\tau\right)$
- sample b_t from $\boldsymbol{q}_t$
- observe loss feedback $\mathbb{I}[a_t \succ b_t]$ (a_t selected by opponent)
- construct estimator $\boldsymbol{\ell}_t \in \mathbb{R}_+^K$ where for each b: $\boldsymbol{\ell}_{t,b} = \frac{\mathbb{I}[b_t = b]\mathbb{I}[a_t \succ b]}{q_{t,b}}$

Losses versus rewards

Exp3 for dueling bandits (**CORRECT** way to select a_t)

For $t = 1, \ldots, T$,

- sample a_t from arm distribution $\boldsymbol{p}_t = \text{softmax}\left(-\eta \sum_{\tau=1}^{t-1} \boldsymbol{\ell}_\tau\right)$
- observe reward feedback $\mathbb{I}[a_t \succ b_t]$ (b_t selected by opponent)
- construct estimator $\boldsymbol{\ell}_t \in \mathbb{R}_+^K$ where for each a: $\boldsymbol{\ell}_{t,a} = \frac{\mathbb{I}[a_t = a]\mathbb{I}[a \prec b_t]}{p_{t,a}}$

- from $\text{softmax}\left(\eta \sum_{\tau=1}^{t-1} \boldsymbol{r}_\tau\right)$ to $\text{softmax}\left(-\eta \sum_{\tau=1}^{t-1} \boldsymbol{\ell}_\tau\right)$
- from $\boldsymbol{r}_{t,a} = \frac{\mathbb{I}[a_t = a]\mathbb{I}[a \succ b_t]}{p_{t,a}}$ to $\boldsymbol{\ell}_{t,a} = \frac{\mathbb{I}[a_t = a]\mathbb{I}[a \prec b_t]}{p_{t,a}}$

How to find Nash Equilibra of a zero-sum game?

Even for games *as large as poker*, **can approximately find one via self-play and regret minimization!**

Self-play for zero-sum games

Input: multi-armed bandit algorithms $\mathcal{A}$ and $\mathcal{B}$
For $t = 1, \ldots, T$,

- get arm distributions $\boldsymbol{p}_t$ and $\boldsymbol{q}_t$ from $\mathcal{A}$ and $\mathcal{B}$ respectively
- sample a_t from $\boldsymbol{p}_t$ and b_t from $\boldsymbol{q}_t$
- observe M_{a_t,b_t} (plus noise), feed it as reward to $\mathcal{A}$ and as loss to $\mathcal{B}$

Low regret $\Rightarrow$ convergence to NE

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2. Basics of Reinforcement learning
   - Markov decision process
   - Learning MDPs
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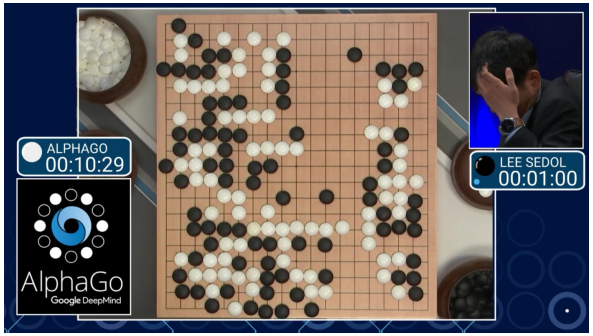
Recent Successes of Deep Reinforcement Learning (RL)

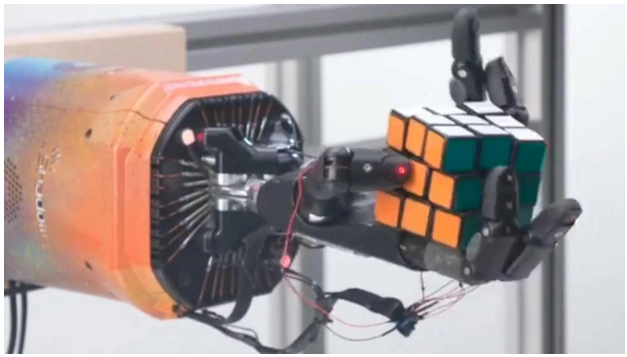
Atari (2013)



Go (2015)

Dota 2 (2017)

StarCraft (2019)



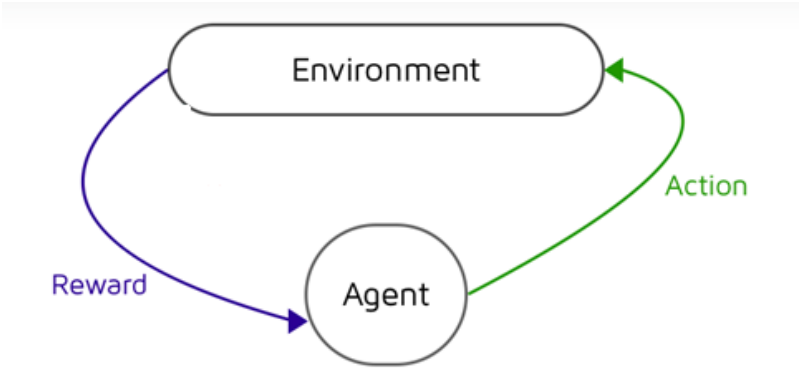
Rubik's Cube (2019)

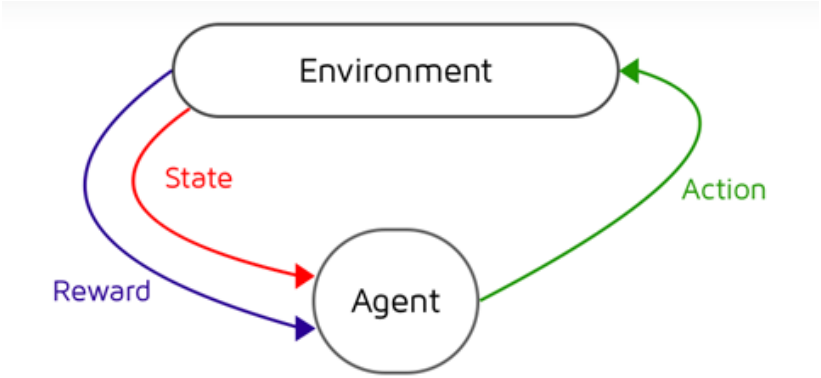
ChatGPT (2022)

Deep RL = RL + deep neural net models, *so what really is RL?*

Motivation

Multi-armed bandit is among the simplest decision making problems with limited feedback.





It's often too simple to capture many real-life problems. One thing it fails to capture is the "state" of the learning agent, which has impacts on the reward of each action.

- e.g. for Atari games, after making one move, the agent moves to a different state, with possible different rewards for each action

Reinforcement learning

Reinforcement learning (RL) is one way to deal with this issue.

The foundation of RL is **Markov Decision Process (MDP)**,
a combination of Markov model (Lec 8) and multi-armed bandit (Lec 10)

Markov Decision Processes (MDPs)

An MDP is parameterized by five elements

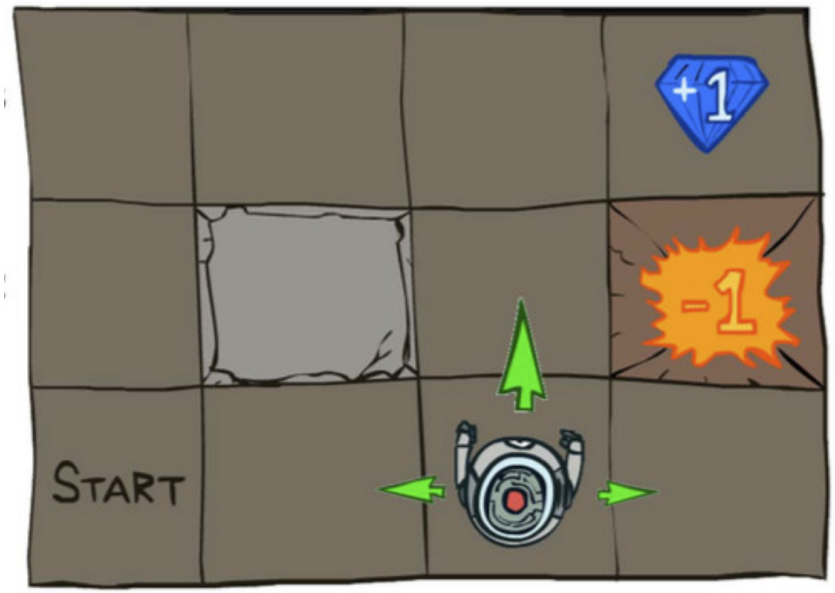
- $\mathcal{S}$: a set of possible states
- $\mathcal{A}$: a set of possible actions
- P: transition probability, $P(s'|s,a)$ is the probability of transiting from state s to state s' after taking action a (Markov property)
- r: reward function, $r(s,a)$ is (expected) reward of action a at state s
- $\gamma \in (0,1]$: discount factor, informally, 1 dollar tomorrow is only worth γ when viewed from today (inflation)

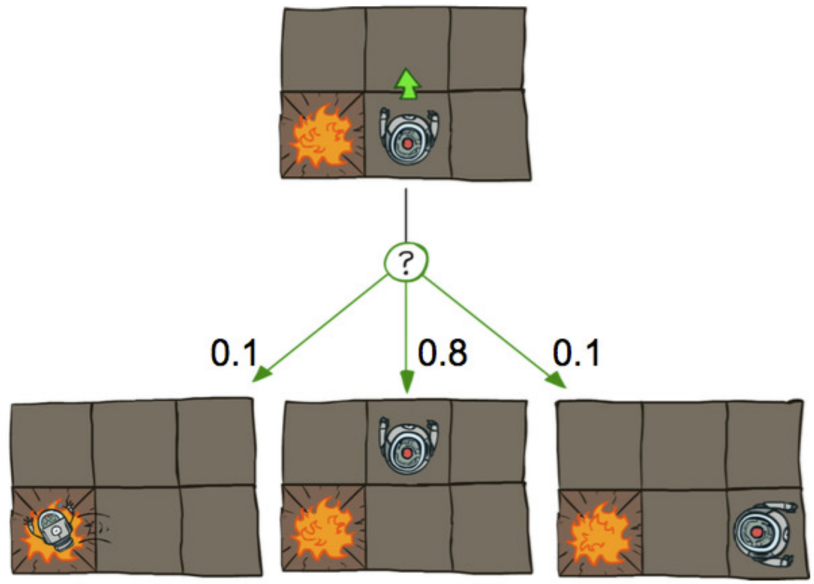
Different from simple Markov chains, the state transition is influenced by the taken action.

Different from Multi-armed bandit, the reward depends on the state.

Example

Canonical example: a grid world





transition model P

- each grid is a state
- 4 actions: up, down, left, right
- reward is 1 for diamond, -1 for fire, and 0 everywhere else

Policy

A **policy** π specifies the probability of taking action a at state s as $\pi(a|s)$.

If we start from state $s_1 \in \mathcal{S}$ and act according to a policy π, the discounted rewards for time $1, 2, \ldots$ are respectively

$$r(s_1, a_1), \quad \gamma r(s_2, a_2), \quad \gamma^2 r(s_3, a_3), \quad \cdots$$

where $a_t \sim \pi(\cdot|s_t)$ and $s_{t+1} \sim P(\cdot|s_t, a_t)$

If we follow the policy forever, the total (discounted) reward is

$$\mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^{t-1} r(s_t, a_t)\right]$$

Optimal Policy and Bellman Equation

First goal: knowing all parameters, *how to find the optimal policy*

$$\operatorname*{argmax}_{\pi} \ \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^{t-1} r(s_t, a_t)\right] \quad ?$$

We first answer a related question: *what is the maximum reward one can achieve starting from an arbitrary state s?*

$$\begin{aligned} V(s) &= \max_{\pi} \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^{t-1} r(s_t, a_t) \,\middle|\, s_1 = s\right] \\ &= \max_{a \in \mathcal{A}} \left(r(s,a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s,a) V(s') \right) \end{aligned}$$

V is called the **optimal value function**. It satisfies the above **Bellman equation**: $|\mathcal{S}|$ nonlinear equations with $|\mathcal{S}|$ unknowns, *how to solve it?*

Value Iteration

Value Iteration

Initialize $V_1(s) = 0$ for all $s \in \mathcal{S}$

For $k = 1, 2, \ldots$ (until convergence), perform **Bellman update**:

$$V_{k+1}(s) \leftarrow \max_{a \in \mathcal{A}} \left(r(s,a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s,a) V_k(s') \right), \quad \forall s \in \mathcal{S}$$

Value iteration converges *exponentially fast*! (HW4)

Knowing V, the optimal policy π^* is simply

$$\pi^*(s) = \operatorname*{argmax}_{a \in \mathcal{A}} \left(r(s,a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s,a) V(s') \right)$$

Learning MDPs

Now suppose we do not know the parameters of the MDP

- transition probability P
- reward function r

How do we find the optimal policy?

- model-based approaches
- model-free approaches

Model-Based Approaches

Key idea: learn the model P and r explicitly from samples

Suppose we have a sequence of interactions: $s_1, a_1, r_1, \ldots, s_T, a_T, r_T$, then the MLE for P and r are simply

$$P(s'|s,a) \propto \#\text{transitions from } s \text{ to } s' \text{ after taking action } a$$
$$r(s,a) = \text{average observed reward at state } s \text{ after taking action } a$$

Having estimates of the parameters we can then apply value iteration to find the optimal policy.

Model-Based Approaches

How do we collect data $s_1, a_1, r_1, s_2, a_2, r_2, \ldots, s_T, a_T, r_T$?

Let's adopt the ϵ-Greedy idea again to ensure exploration.

A sketch for model-based approaches

Initialize V

For $t = 1, 2, \ldots,$

- **with probability** ϵ**, explore**: pick an action uniformly at random
- **with probability** $1 - \epsilon$**, exploit**: pick the optimal action based on V
- update the model parameters P, r
- update the value function V (via value iteration)

Model-Free Approaches

Key idea: do not learn the model explicitly. *What do we learn then?*

Define the $Q : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ function as

$$Q(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s,a) \max_{a' \in \mathcal{A}} Q(s',a')$$

In words, $Q(s,a)$ is the expected reward one can achieve starting from state s with action a, then acting optimally.

Clearly, $V(s) = \max_a Q(s,a)$.

Knowing $Q(s,a)$, the optimal policy at state s is simply $\operatorname{argmax}_a Q(s,a)$.

Model-free approaches learn the Q function directly from samples.

Temporal Difference (TD error)

How to learn the Q function?

$$Q(s,a) = r(s,a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s,a) \max_{a' \in \mathcal{A}} Q(s',a')$$

Given experience (s_t, a_t, r_t, s_{t+1}), with the current guess on Q, $y_t = r_t + \gamma \max_{a'} Q(s_{t+1}, a')$ is like a sample of the RHS of the equation.

So it's natural to do the following update (with learning rate α):

$$\begin{aligned} Q(s_t,a_t) &\leftarrow (1-\alpha)Q(s_t,a_t) + \alpha y_t \\ &= Q(s_t,a_t) + \alpha \underbrace{(y_t - Q(s_t,a_t))}_{\text{temporal difference}} \\ &= Q(s_t,a_t) - \alpha \frac{\partial \left(\frac{1}{2} \left(Q(s_t,a_t) - y_t \right)^2 \right)}{\partial Q(s_t,a_t)} \end{aligned}$$

which is gradient descent w.r.t. squared loss $\frac{1}{2} \left(Q(s_t,a_t) - y_t \right)^2$.

Q-learning

The simplest model-free algorithm:

Q-learning

Initialize Q

For $t = 1, 2, \ldots,$

- **with probability ϵ, explore**: a_t is chosen uniformly at random
- **with probability $1 - \epsilon$, exploit**: $a_t = \operatorname{argmax}_a Q(s_t, a)$
- execute action a_t, receive reward r_t, arrive at state s_{t+1}
- update the Q function

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) - \alpha \left(Q(s_t, a_t) - r_t - \gamma \max_a Q(s_{t+1}, a) \right)$$

for some learning rate α.

Comparisons

	Model-based	Model-free
What it learns	model parameters $P, r, \ldots$	Q function
Space	$O(\lvert\mathcal{S}\rvert^2\lvert\mathcal{A}\rvert)$	$O(\lvert\mathcal{S}\rvert\lvert\mathcal{A}\rvert)$
Sample efficiency	usually better	usually worse

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Function approximation

Algorithms discussed so far (called **tabular algorithms**) run in time/space poly($|\mathcal{S}||\mathcal{A}|$), which is impractical. (Go has about 2×10^{170} states!)

To overcome this issue, we approximate Q by a function parametrized by θ:

$$Q_\theta(s,a) \approx Q(s,a),\ \forall\ (s,a)$$

- (simplest) linear function approximation: $Q_\theta(s,a) = \langle \theta, \phi(s,a) \rangle$ for some "feature" $\phi(s,a)$
- **deep Q-network** (DQN): Q_θ is a neural net with weight θ

Q-learning with function approximation

How to learn θ?

Recall in the tabular case, with $y_t = r_t + \gamma \max_{a'} Q(s_{t+1}, a')$:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \underbrace{(y_t - Q(s_t, a_t))}_{\text{temporal difference}}$$

$$= Q(s_t, a_t) - \alpha \frac{\partial \left(\frac{1}{2} \left(Q(s_t, a_t) - y_t \right)^2 \right)}{\partial Q(s_t, a_t)}$$

A natural generalization: perform gradient descent on θ with squared loss $\frac{1}{2} \left(Q_\theta(s_t, a_t) - y_t \right)^2$:

$$\theta \leftarrow \theta - \alpha \nabla_\theta \left(\frac{1}{2} \left(Q_\theta(s_t, a_t) - y_t \right)^2 \right)$$

$$= \theta - \alpha \left(Q_\theta(s_t, a_t) - y_t \right) \nabla_\theta Q_\theta(s_t, a_t)$$

Q-learning with function approximation

Q-learning

Initialize θ randomly

For $t = 1, 2, \ldots,$

- **with probability ϵ, explore**: a_t is chosen uniformly at random
- **with probability $1 - \epsilon$, exploit**: $a_t = \operatorname{argmax}_a Q_\theta(s_t, a)$
- execute action a_t, receive reward r_t, arrive at state s_{t+1}
- update the parameter of the Q function

$$\theta \leftarrow \theta - \alpha \left(Q_\theta(s_t, a_t) - y_t\right) \nabla_\theta Q_\theta(s_t, a_t)$$

for some learning rate α.

Case study: superhuman AI for Atari games [Deepmind, 2013]

Model each Atari game as an MDP $(\mathcal{S}, \mathcal{A}, P, r, \gamma)$:

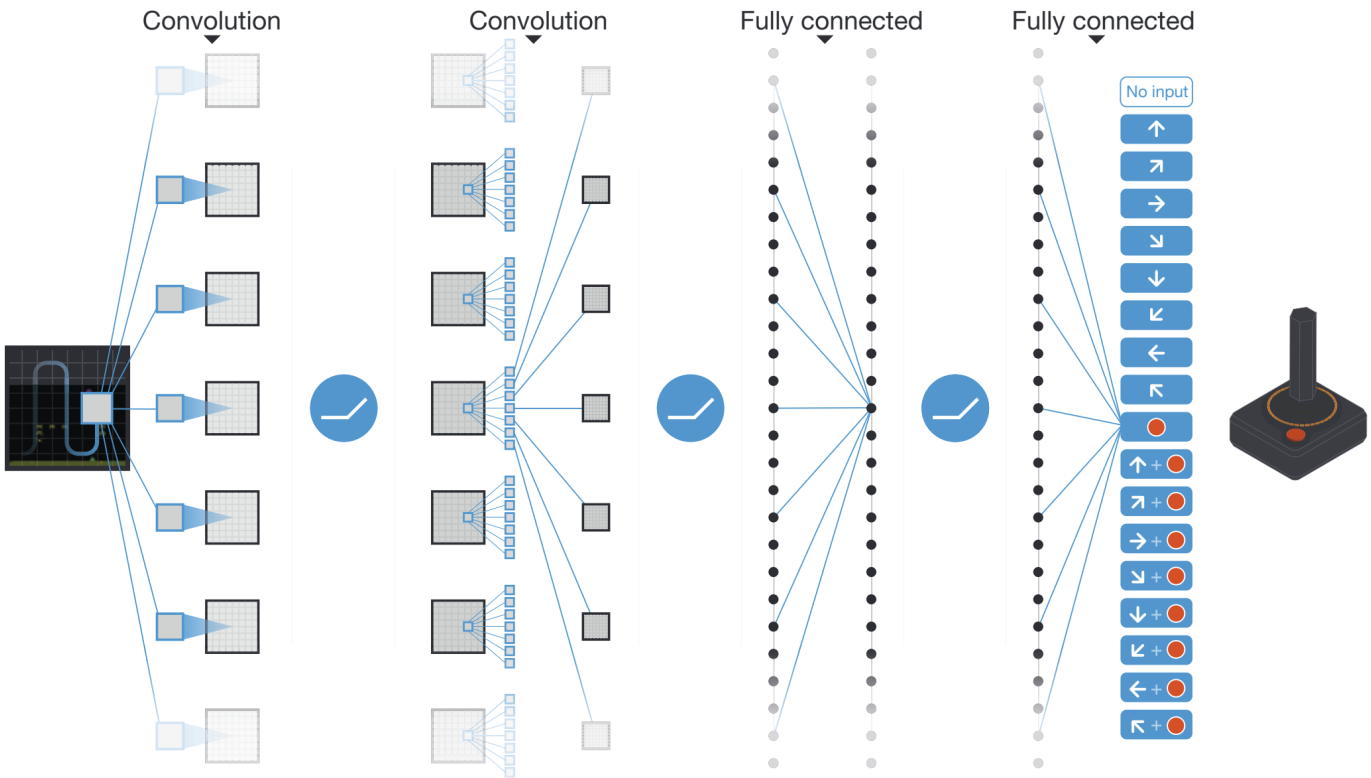
- states: **raw images** (84×84 after preprocessing)
  - no feature engineering, end-to-end (from pixel to action) reinforcement learning, just like humans
  - stack 4 most recent frames as one state (to make things Markovian)

- 18 possible actions:

- transition: determined by each game
- reward: change in score
- $\gamma = 0.99$ (but note that the game will end at some point)

Deep Q-Network

- input: $84 \times 84 \times 4$ images
- 3 convolutional layers $+$ 2 fully-connected layers, 3M parameters
- each of the 18 outputs specifies the Q-value of the corresponding action given a certain state input



Training

For each game, run Q-learning for $T = 50\text{M}$ (around 38 days of game experience), with **two more tricks**:

- use a target network $\bar{\theta}$ to stabilize training

$$y_t = r_t + \gamma \max_{a'} Q_\theta(s_{t+1}, a') \implies y_t = r_t + \gamma \max_{a'} Q_{\bar{\theta}}(s_{t+1}, a')$$

  - $\bar{\theta}$ is a snapshot of θ, updated every 10K rounds

- use experience replay to reduce correlation / increase data efficiency
  - instead of using one sample in each update, use a minibatch of 32 samples randomly selected from the most recent 1M frames

$$(Q_\theta(s_t, a_t) - y_t)^2 \implies \sum_{k \in \text{minibatch}} (Q_\theta(s_k, a_k) - y_k)^2$$

More on experience replay

Use a minibatch of samples from previous experience

- target: from $(Q_\theta(s_t, a_t) - y_t)^2$ to $\sum_{k \in \text{minibatch}} (Q_\theta(s_k, a_k) - y_k)^2$
- update: from

$$\theta \leftarrow \theta - \alpha \left(Q_\theta(s_t, a_t) - y_t\right) \nabla_\theta Q_\theta(s_t, a_t)$$

to

$$\theta \leftarrow \theta - \alpha \sum_{k \in \text{minibatch}} \left(Q_\theta(s_k, a_k) - y_k\right) \nabla_\theta Q_\theta(s_k, a_k)$$

- in the tabular case, it means from (see programming project)

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) - \alpha(Q(s_t, a_t) - y_t)$$

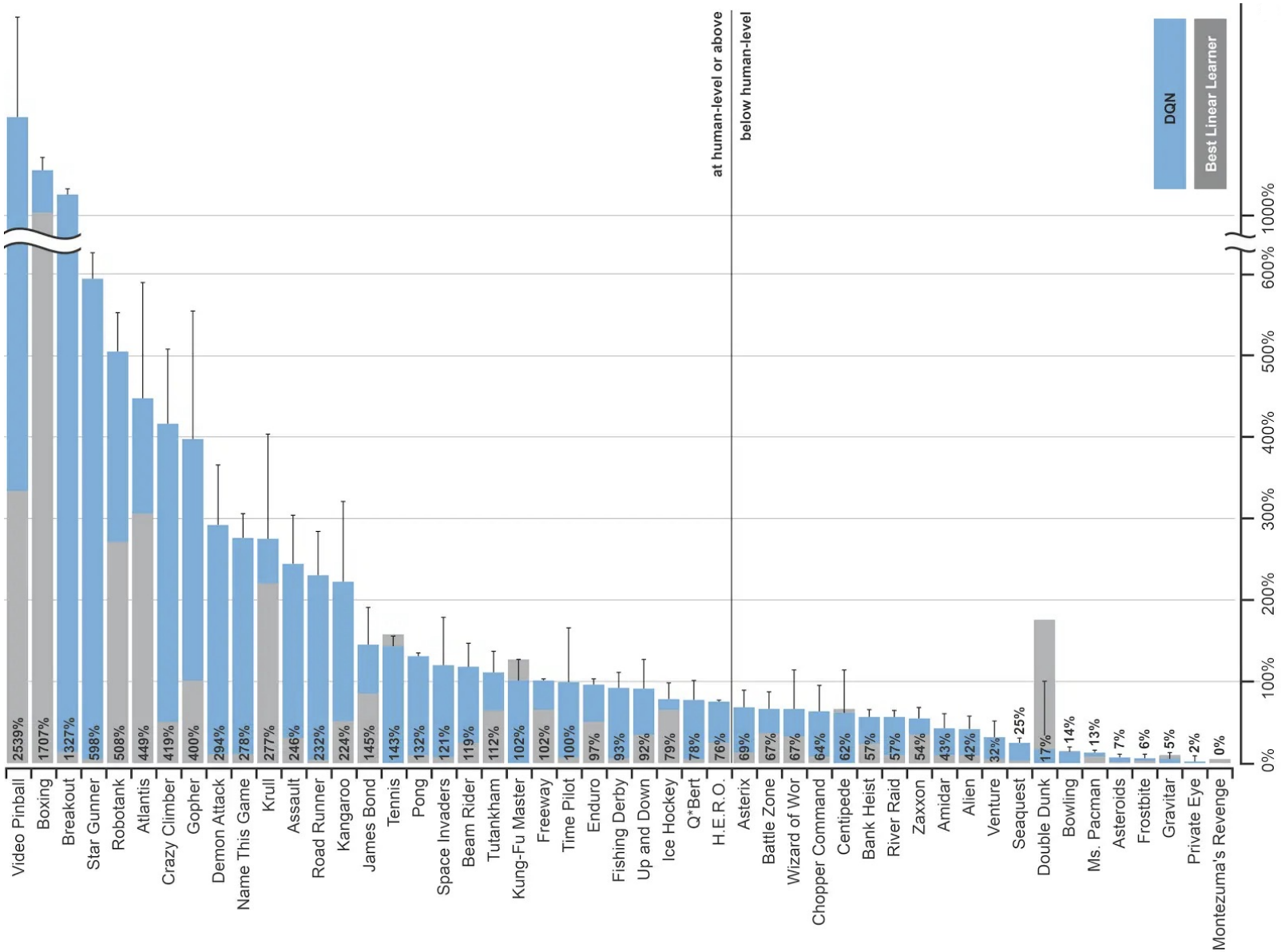
to

$$Q(s_k, a_k) \leftarrow Q(s_k, a_k) - \alpha(Q(s_k, a_k) - y_k), \quad \forall k \in \text{minibatch}$$

Results

- tested on 49 Atari Games, 5 mins each game for 30 times
- same model architecture, same algorithm, same hyperparameters
- compared against best linear learner and a professional human tester
- report $\frac{\text{DQN score} - \text{random play score}}{\text{human score} - \text{random play score}} \times 100\%$

Results


DQN
Best Linear Learner
at human-level or above
below human-level
0%
100%
200%
300%
400%
500%
600%
1000%
Video Pinball 2539%
Boxing 1707%
Breakout 1327%
Star Gunner 598%
Robotank 508%
Atlantis 449%
Crazy Climber 419%
Gopher 400%
Demon Attack 294%
Name This Game 278%
Krull 277%
Assault 246%
Road Runner 232%
Kangaroo 224%
James Bond 145%
Tennis 143%
Pong 132%
Space Invaders 121%
Beam Rider 119%
Tutankham 112%
Kung-Fu Master 102%
Freeway 102%
Time Pilot 100%
Enduro 97%
Fishing Derby 93%
Up and Down 92%
Ice Hockey 79%
Q*Bert 78%
H.E.R.O. 76%
Asterix 69%
Battle Zone 67%
Wizard of Wor 67%
Chopper Command 64%
Centipede 62%
Bank Heist 57%
River Raid 57%
Zaxxon 54%
Amidar 43%
Alien 42%
Venture 32%
Seaquest 25%
Double Dunk 17%
Bowling 14%
Ms. Pacman 13%
Asteroids 7%
Frostbite 6%
Gravitar 5%
Private Eye 2%
Montezuma's Revenge 0%

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Learning policies directly

Another popular class of RL algorithms learns the policy directly:

$$\max_{\pi} \text{ “expected reward of policy } \pi\text{”}$$

To handle large scale problems, consider a parameterized policy class $\Pi = \{\pi_\rho : \rho \in \Omega\}$ (e.g., a set of neural nets) and solve

$$\max_{\rho \in \Omega} \text{ “expected reward of policy } \pi_\rho\text{”}$$

via stochastic gradient descent

Policy gradient theorem

For simplicity, suppose $\gamma = 1$ and a trajectory ends after H steps.

Expected reward of π_ρ can be written as

$$R(\pi_\rho) = \sum_\tau P_\rho(\tau) R(\tau)$$

- $\tau = (s_1, a_1, \ldots, s_H, a_H)$ ranges over all possible H-step trajectories
- $P_\rho(\tau)$ is the probability of encountering trajectory τ under policy π_ρ
- $R(\tau) = \sum_{h=1}^{H} r(s_h, a_h)$ is the cumulative reward for trajectory τ

So we have

$$\nabla_\rho R(\pi_\rho) = \sum_\tau \nabla_\rho P_\rho(\tau) R(\tau)$$

How do we efficiently compute/approximate it?

Policy gradient theorem (cont.)

$$\nabla_\rho R(\pi_\rho) = \sum_\tau \nabla_\rho P_\rho(\tau) R(\tau) = \sum_\tau P_\rho(\tau) \frac{\nabla_\rho P_\rho(\tau)}{P_\rho(\tau)} R(\tau)$$

$$= \sum_\tau P_\rho(\tau) \nabla_\rho \log P_\rho(\tau) R(\tau) \qquad \text{(log derivative trick)}$$

$$= \mathbb{E}_\tau \left[\nabla_\rho \log P_\rho(\tau) R(\tau) \right] \qquad \text{(written as an } \textbf{expectation}\text{)}$$

$$= \mathbb{E}_\tau \left[\nabla_\rho \log \left(\Pi_{h=1}^H \pi_\rho(a_h|s_h) P(s_{h+1}|s_h, a_h) \right) R(\tau) \right]$$

$$= \mathbb{E}_\tau \left[\left(\sum_{h=1}^H \nabla_\rho \log \pi_\rho(a_h|s_h) \right) R(\tau) \right] \qquad \text{(}\textit{transition doesn't matter!}\text{)}$$

which can be approximated by sampling n trajectories using π_ρ and taking the empirical average:

$$\frac{1}{n} \sum_{i=1}^n \left(\sum_{h=1}^H \nabla_\rho \log \pi_\rho(a_h^{(i)}|s_h^{(i)}) \right) R(\tau^{(i)})$$

Reducing variance of gradient estimators via baselines

The key to make policy gradient work is to **reduce variance** of gradient estimators. Subtracting a "baseline" is a standard way to achieve so:

$$\nabla_\rho R(\pi_\rho) = \mathbb{E}_\tau \left[\sum_{h=1}^{H} \nabla_\rho \log \pi_\rho(a_h|s_h) R(\tau) \right]$$

$$= \mathbb{E}_\tau \left[\sum_{h=1}^{H} \nabla_\rho \log \pi_\rho(a_h|s_h) \left(R(\tau) - b(s_{1:h}, a_{1:h-1}) \right) \right]$$

This holds *for any b that only depends on $s_{1:h}, a_{1:h-1}$*, because

$$\mathbb{E}_{a_h} \left[\nabla_\rho \log \pi_\rho(a_h|s_h) b \right] = b \sum_{a_h \in \mathcal{A}} \pi_\rho(a_h|s_h) \frac{\nabla_\rho \pi_\rho(a_h|s_h)}{\pi_\rho(a_h|s_h)}$$

$$= b \nabla_\rho \sum_{a_h \in \mathcal{A}} \pi_\rho(a_h|s_h) = b \nabla_\rho 1 = 0$$

Which baselines?

$$\nabla_\rho R(\pi_\rho) = \mathbb{E}_\tau \left[\sum_{h=1}^{H} \nabla_\rho \log \pi_\rho(a_h|s_h) \left(R(\tau) - b(s_{1:h}, a_{1:h-1}) \right) \right]$$

Want $b(s_{1:h}, a_{1:h-1})$ to be close to $R(\tau)$, leading to an **idealized** choice:

"observed reward before h" + "expected reward starting from h"

$$= \left(\sum_{h'=1}^{h-1} r(s_{h'}, a_{h'}) \right) + \underbrace{\mathbb{E} \left[\sum_{h'=h}^{H} r(s_{h'}, a_{h'}) \mid s_{h'} = s_h \right]}_{V_{\pi_\rho}(s_h)}$$

V_{π_ρ}, called a **critic**, is usually **approximated** by another network θ:

"observed reward before h" + "estimated reward starting from h"

$$= \left(\sum_{h'=1}^{h-1} r(s_{h'}, a_{h'}) \right) + V_\theta(s_h)$$

Actor-Critic methods

Repeat:

- Critic evaluates the current policy π_ρ by fitting V_θ from samples using square loss:

$$\min_\theta \sum_{j=1}^{m} \sum_{h=1}^{H} \left(V_\theta \left(s_h^{(j)} \right) - \sum_{h'=h}^{H} r \left(s_{h'}^{(j)}, a_{h'}^{(j)} \right) \right)^2$$

- Actor improves the current policy π_ρ via stochastic gradient descent:

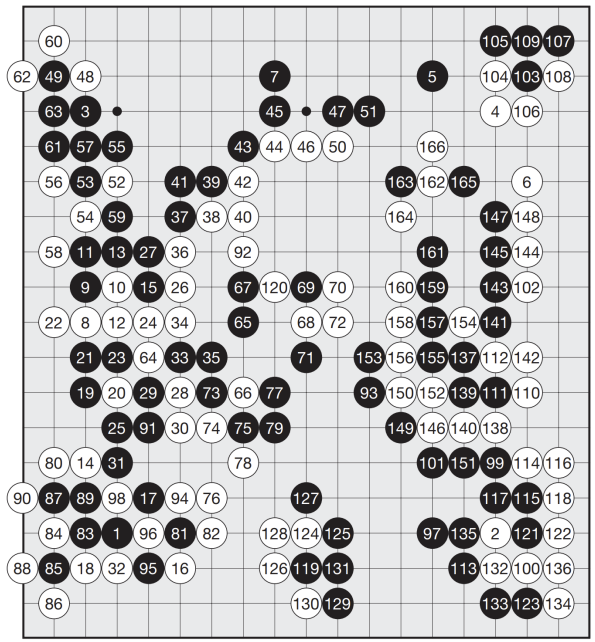
$$\rho \leftarrow \rho - \frac{\alpha}{n} \sum_{i=1}^{n} \sum_{h=1}^{H} \nabla_\rho \log \pi_\rho(a_h^{(i)} | s_h^{(i)}) \underbrace{\left(\sum_{h'=h}^{H} r \left(s_{h'}^{(i)}, a_{h'}^{(i)} \right) - V_\theta(s_h^{(i)}) \right)}_{= R(\tau^{(i)}) - b(s_{1:h}^{(i)}, a_{1:h-1}^{(i)})}$$

Case study: AlphaGo

[Deepmind, 2015]

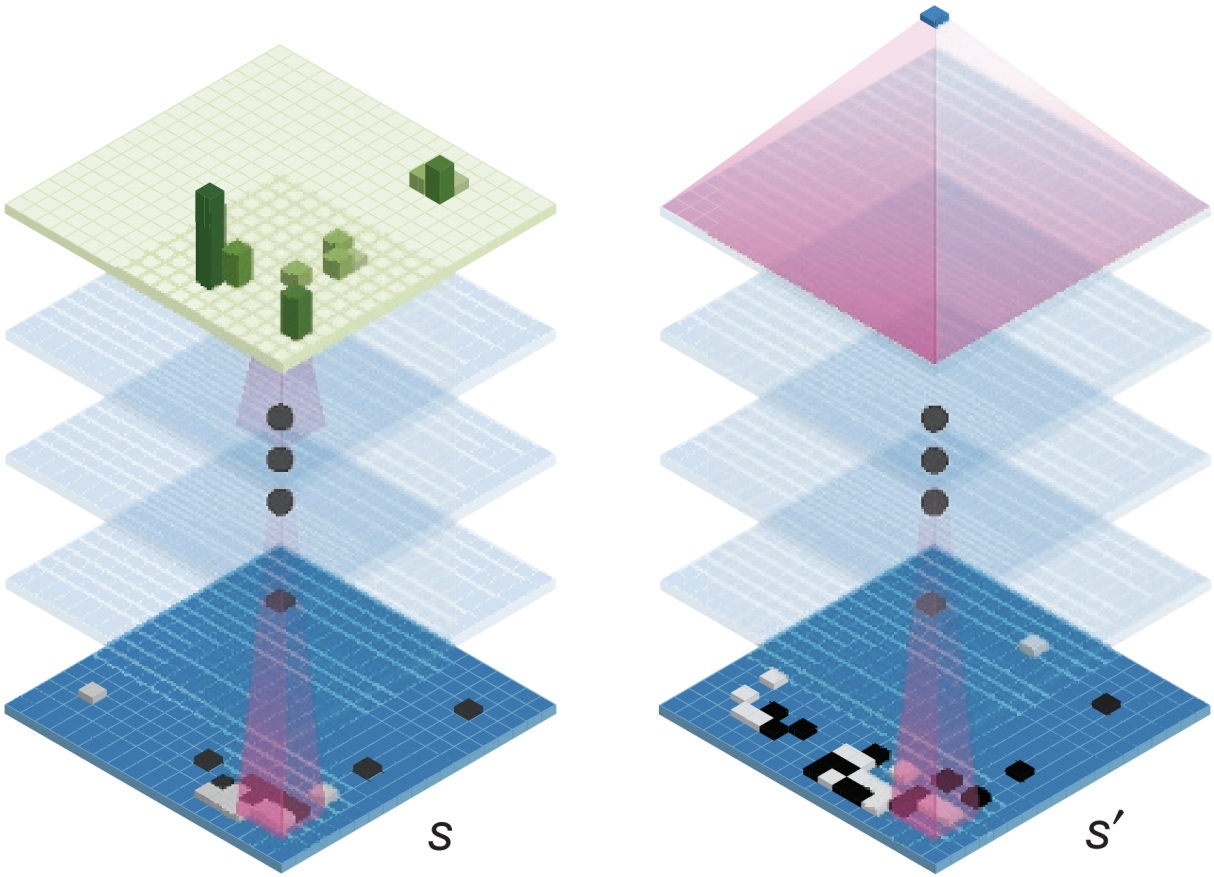
Model Go as an MDP $(\mathcal{S}, \mathcal{A}, P, r, \gamma)$:

- states: each 19×19 position of the game is pre-processed into an $19 \times 19 \times 48$ image stack consisting of feature planes
- actions: all legal next moves
- transition: determined by the opponent
- reward: only the ending state has reward (1 if win, -1 if lose)
- $\gamma = 1$



Policy/value networks

Both π_ρ and V_θ are large convolutional neural nets:



Training

Step 1: first train a policy π_σ using pure **supervised learning** from 30M expert moves (a multiclass classification task)

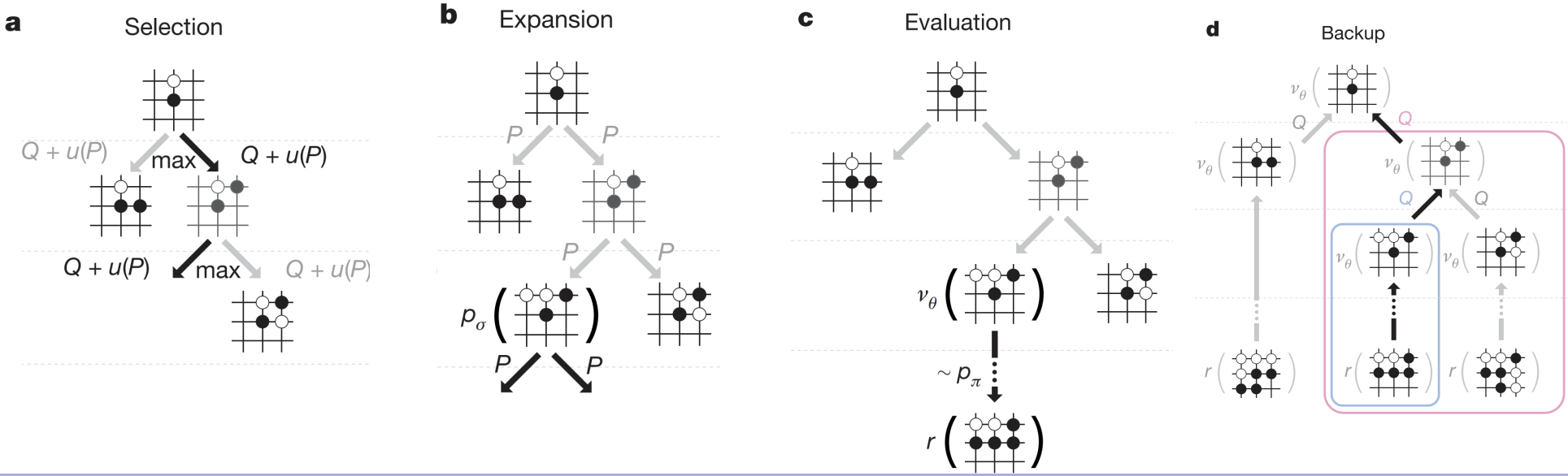
Step 2: use **actor-critic** to train policy network π_ρ and value network V_θ

- initialize ρ as σ
- self-play: every 500 iterations, add current ρ to an opponent pool; in each iteration, randomly sampled one from this pool as the opponent
- trained for 10K iterations, each with 128 games

Testing (actual play)

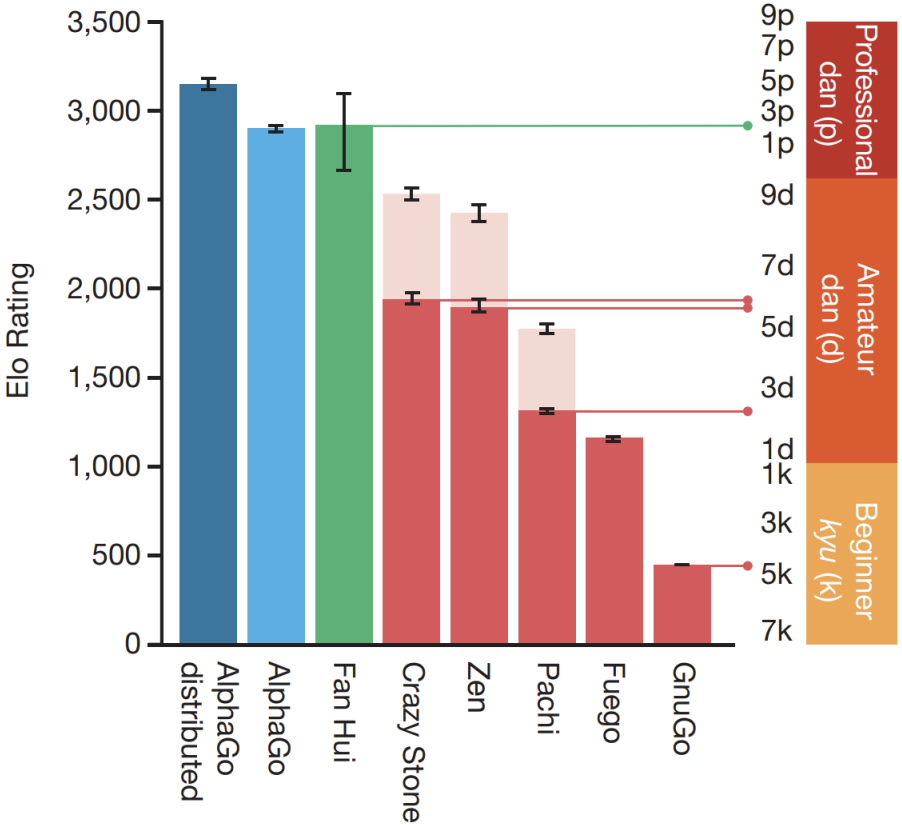
"**Monte-Carlo Tree Search**" with the help of policy/value networks:

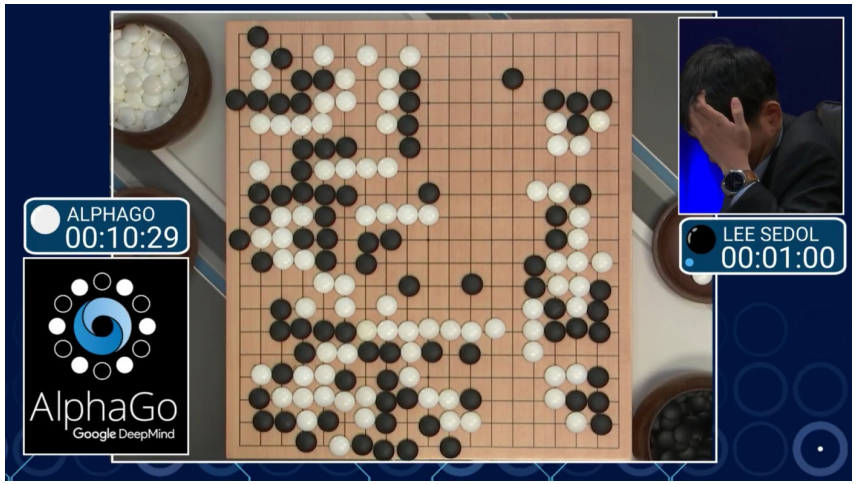
- **select** a move with highest estimated quality Q + UCB (inversely proportional to #visits, just like bandits)
- **expand** a leaf (when visited more than a certain times) using π_σ
- **evaluate** the leaf by averaging V_θ's prediction and a random rollout
- **update** the quality Q value along the traversed edges
- when the search halts, select the most visited move at the root



Results

- 99.8% win rate against other Go programs
- 5-0 Fan Hui (2013/2014/2015 European Go champion)
- first superhuman AI for Go, previously believed to be a decade away





Summary

A brief introduction to (deep) RL:

- **foundation**: MDP, value iteration, model-based/free learning
- **large-scale and practical** deep RL methods:
  - Q-learning with function approximation, DQN, and their success in Atari games
  - policy gradient, actor-critic methods, and their success in Go