CSCI567 Machine Learning (Fall 2025)

Haipeng Luo

University of Southern California

Nov 21, 2025

Exam 2 Logistics

Date: Friday, Dec 5th

Time: 2:00-4:00pm (plus another 20 mins for uploading)

Location: THH 201 (Initial A-R) and SGM 101 (Initial S-Z)

Individual effort, close-book (no cheat sheet), no calculators or any other electronics, *but need your phone to upload your solutions to Gradescope from 4:00-4:20pm*

Exam 2 Coverage

Coverage: mostly Lec 7-11 (just see the sample)

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Seven problems in total

- one problem of 15 multiple-choice multiple-answer questions
  - *please note the new instructions!!*
- six other homework-like problems, each has a couple sub-problems
  - clustering, EM, HMM, RNN/transformer, bandits, RL

Outline

1. Review of last lecture
2. Basics of Reinforcement learning
3. Deep Q-Networks and Atari Games
4. Policy Gradient, Actor-Critic, and AlphaGo

Outline

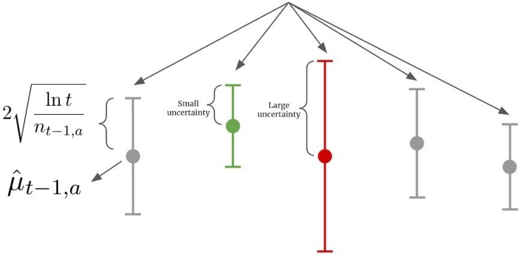
UCB for multi-armed bandits

Adaptive exploration-exploitation trade-off via optimism

Upper Confidence Bound (UCB) algorithm

For $t = 1, \ldots, T$, pick $a_t = \mathrm{argmax}_a \ \mathrm{UCB}_{t,a}$ where

$$\mathrm{UCB}_{t,a} \triangleq \hat{\mu}_{t-1,a} + 2\sqrt{\frac{\ln t}{n_{t-1,a}}}$$



Self-play for dueling bandits (preference feedback)

Exp3 for dueling bandits (selecting b_t)

Input: a learning rate parameter $\eta > 0$

For $t = 1, \ldots, T$,

- compute arm distribution $\boldsymbol{q}_t = \text{softmax}\left(-\eta \sum_{\tau=1}^{t-1} \boldsymbol{\ell}_\tau\right)$
- sample b_t from $\boldsymbol{q}_t$
- observe loss feedback $\mathbb{I}[a_t \succ b_t]$ (a_t selected by opponent)
- construct estimator $\boldsymbol{\ell}_t \in \mathbb{R}_+^K$ where for each b: $\boldsymbol{\ell}_{t,b} = \frac{\mathbb{I}[b_t = b]\mathbb{I}[a_t \succ b]}{q_{t,b}}$

Losses versus rewards

Exp3 for dueling bandits (CORRECT way to select a_t)

For $t = 1, \ldots, T$,

- sample a_t from arm distribution $\boldsymbol{p}_t = \text{softmax}\left(-\eta \sum_{\tau=1}^{t-1} \boldsymbol{\ell}_\tau\right)$
- observe reward feedback $\mathbb{I}[a_t \succ b_t]$ (b_t selected by opponent)
- construct estimator $\boldsymbol{\ell}_t \in \mathbb{R}_+^K$ where for each a: $\boldsymbol{\ell}_{t,a} = \frac{\mathbb{I}[a_t = a]\mathbb{I}[a \prec b_t]}{p_{t,a}}$

- from $\text{softmax}\left(\eta \sum_{\tau=1}^{t-1} \boldsymbol{r}_\tau\right)$ to $\text{softmax}\left(-\eta \sum_{\tau=1}^{t-1} \boldsymbol{\ell}_\tau\right)$
- from $\boldsymbol{r}_{t,a} = \frac{\mathbb{I}[a_t = a]\mathbb{I}[a \succ b_t]}{p_{t,a}}$ to $\boldsymbol{\ell}_{t,a} = \frac{\mathbb{I}[a_t = a]\mathbb{I}[a \prec b_t]}{p_{t,a}}$

How to find Nash Equilibra of a zero-sum game?

Even for games *as large as poker*, **can approximately find one via self-play and regret minimization!**

Self-play for zero-sum games

Input: multi-armed bandit algorithms $\mathcal{A}$ and $\mathcal{B}$

For $t = 1, \ldots, T$,

- get arm distributions $\boldsymbol{p}_t$ and $\boldsymbol{q}_t$ from $\mathcal{A}$ and $\mathcal{B}$ respectively
- sample a_t from $\boldsymbol{p}_t$ and b_t from $\boldsymbol{q}_t$
- observe M_{a_t,b_t} (plus noise), feed it as reward to $\mathcal{A}$ and as loss to $\mathcal{B}$

Low regret $\Rightarrow$ convergence to NE

Outline

Recent Successes of Deep Reinforcement Learning (RL)

Atari (2013)

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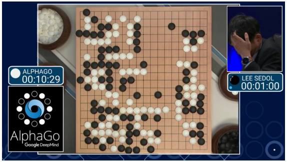

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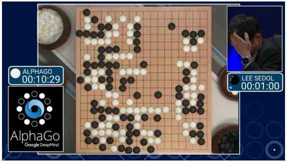

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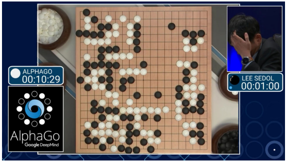

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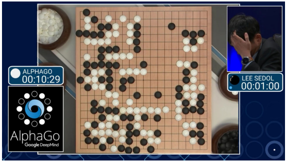

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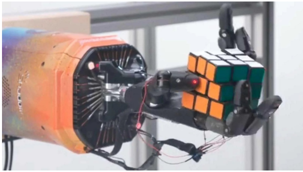

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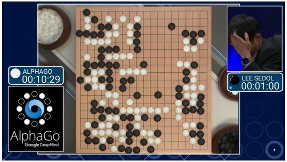

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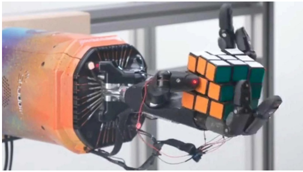

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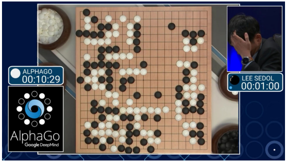

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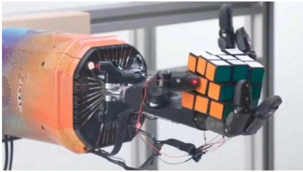

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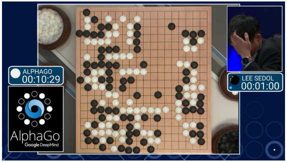

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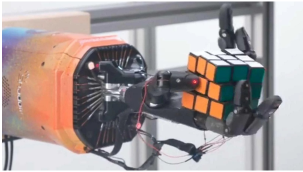

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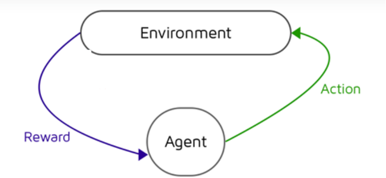
Deep RL = RL + deep neural net models, *so what really is RL?*

Motivation

Multi-armed bandit is among the simplest decision making problems with limited feedback.



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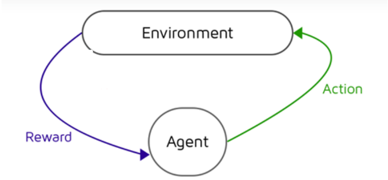

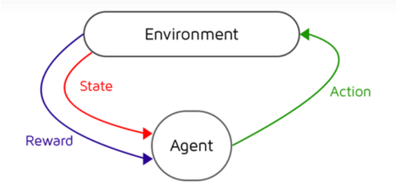

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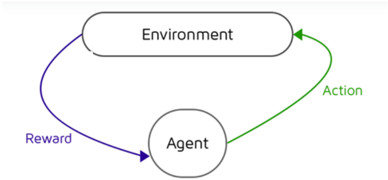

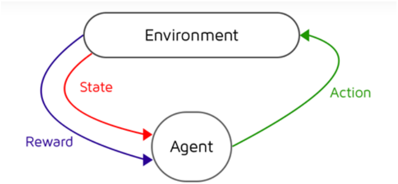

It's often too simple to capture many real-life problems. One thing it fails to capture is the "state" of the learning agent, which has impacts on the reward of each action.

- e.g. for Atari games, after making one move, the agent moves to a different state, with possible different rewards for each action

Reinforcement learning

Reinforcement learning (RL) is one way to deal with this issue.

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The foundation of RL is **Markov Decision Process (MDP)**,
a combination of Markov model (Lec 8) and multi-armed bandit (Lec 10)

Markov Decision Processes (MDPs)

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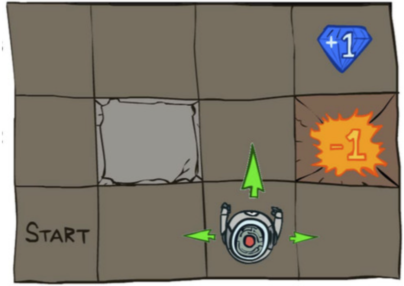
Different from Multi-armed bandit, the reward depends on the state.

Example

Canonical example: a grid world



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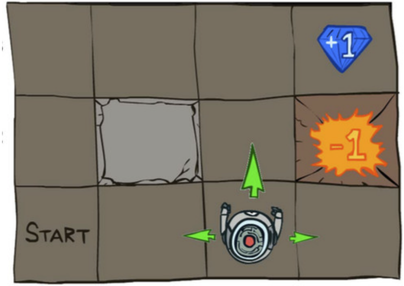

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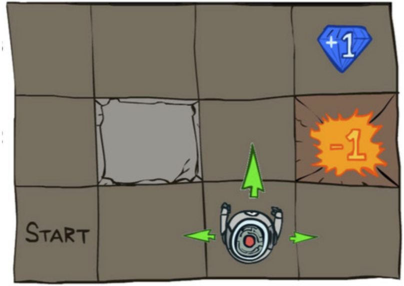

- each grid is a state
- 4 actions: up, down, left, right

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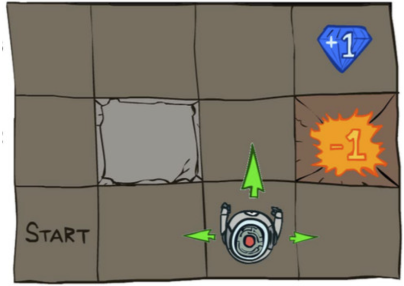

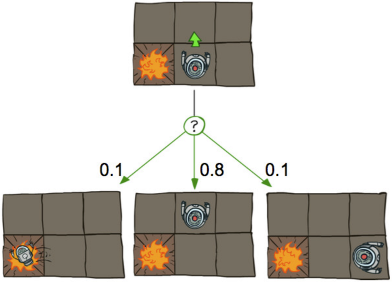

transition model P

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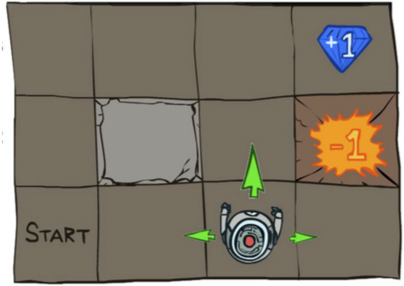

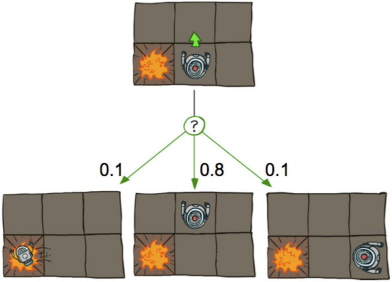

transition model P

- each grid is a state
- 4 actions: up, down, left, right
- reward is 1 for diamond, -1 for fire, and 0 everywhere else

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$$r(s_1, a_1), \;\; \gamma r(s_2, a_2), \;\; \gamma^2 r(s_3, a_3), \;\; \cdots$$

where $a_t \sim \pi(\cdot|s_t)$ and $s_{t+1} \sim P(\cdot|s_t, a_t)$

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If we follow the policy forever, the total (discounted) reward is

$$\mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^{t-1} r(s_t, a_t)\right]$$

Optimal Policy and Bellman Equation

First goal: knowing all parameters, *how to find the optimal policy*

$$\underset{\pi}{\operatorname{argmax}} \ \mathbb{E}\left[\sum_{t=1}^{\infty} \gamma^{t-1} r(s_t, a_t)\right] \quad ?$$

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V is called the **optimal value function**. It satisfies the above **Bellman equation**: $|\mathcal{S}|$ nonlinear equations with $|\mathcal{S}|$ unknowns, *how to solve it?*

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Initialize $V_1(s) = 0$ for all $s \in \mathcal{S}$

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For $k = 1, 2, \ldots$ (until convergence), perform **Bellman update**:

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Knowing V, the optimal policy π^* is simply

$$\pi^*(s) = \operatorname*{argmax}_{a \in \mathcal{A}} \left(r(s,a) + \gamma \sum_{s' \in \mathcal{S}} P(s'|s,a) V(s') \right)$$

Learning MDPs

Now suppose we do not know the parameters of the MDP

- transition probability P
- reward function r

How do we find the optimal policy?

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How do we find the optimal policy?

- model-based approaches
- model-free approaches

Model-Based Approaches

Key idea: learn the model P and r explicitly from samples

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$$P(s'|s,a) \propto \#\text{transitions from } s \text{ to } s' \text{ after taking action } a$$
$$r(s,a) = \text{average observed reward at state } s \text{ after taking action } a$$

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Having estimates of the parameters we can then apply value iteration to find the optimal policy.

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A sketch for model-based approaches

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A sketch for model-based approaches

Initialize V

For $t = 1, 2, \ldots,$

- **with probability ϵ, explore**: pick an action uniformly at random

Model-Based Approaches

How do we collect data $s_1, a_1, r_1, s_2, a_2, r_2, \ldots, s_T, a_T, r_T$?

Let's adopt the ϵ-Greedy idea again to ensure exploration.

A sketch for model-based approaches

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- update the value function V (via value iteration)

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Model-free approaches learn the Q function directly from samples.

Temporal Difference (TD error)

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Sample efficiency	usually better	usually worse

Outline

Function approximation

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- **deep Q-network** (DQN): Q_θ is a neural net with weight θ

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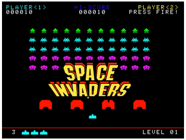

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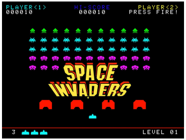

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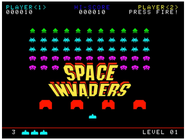

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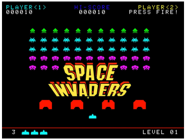

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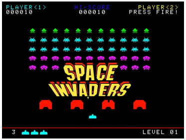

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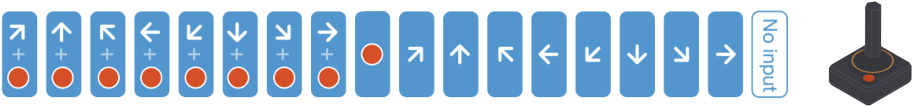

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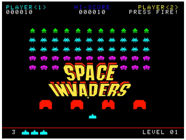

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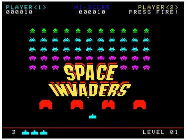

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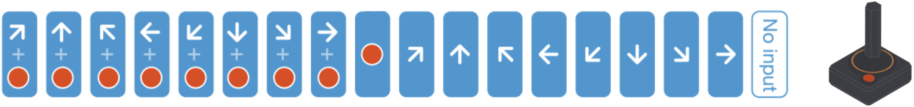

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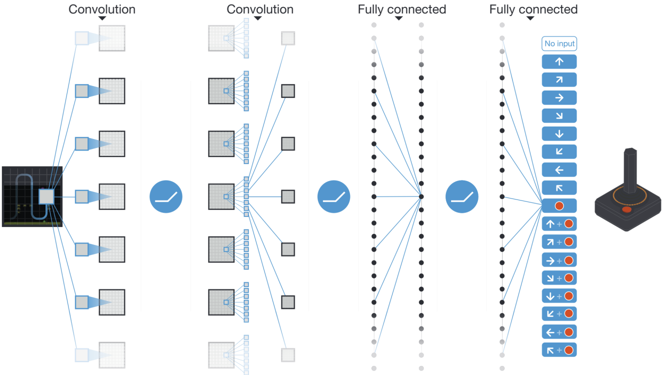
Deep Q-Network



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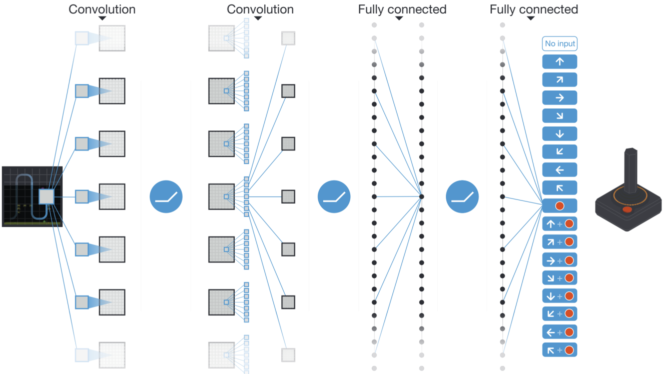

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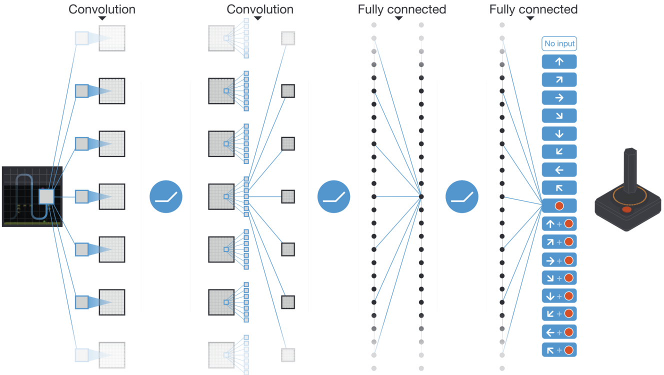

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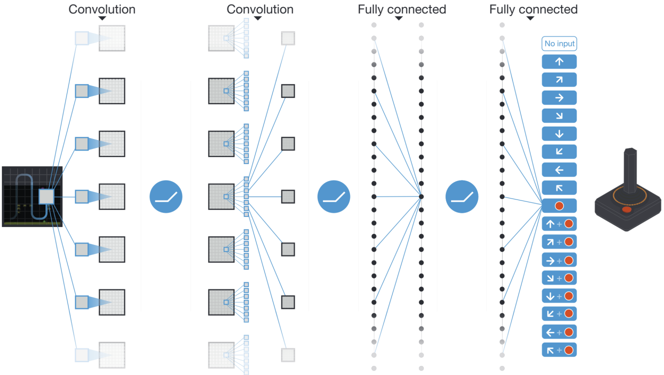

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  - instead of using one sample in each update,

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For each game, run Q-learning for $T = 50\text{M}$ (around 38 days of game experience), with **two more tricks**:

- use a target network $\bar{\theta}$ to stabilize training

$$y_t = r_t + \gamma \max_{a'} Q_\theta(s_{t+1}, a') \implies y_t = r_t + \gamma \max_{a'} Q_{\bar{\theta}}(s_{t+1}, a')$$

  - $\bar{\theta}$ is a snapshot of θ, updated every 10K rounds

- use experience replay to reduce correlation / increase data efficiency
  - instead of using one sample in each update, use a minibatch of 32 samples randomly selected from the most recent 1M frames

$$(Q_\theta(s_t, a_t) - y_t)^2 \implies \sum_{k \in \text{minibatch}} (Q_\theta(s_k, a_k) - y_k)^2$$

More on experience replay

Use a minibatch of samples from previous experience

- target: from $(Q_\theta(s_t, a_t) - y_t)^2$ to $\sum_{k \in \text{minibatch}} (Q_\theta(s_k, a_k) - y_k)^2$

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- in the tabular case, it means from (see programming project)

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) - \alpha(Q(s_t, a_t) - y_t)$$

to

$$Q(s_k, a_k) \leftarrow Q(s_k, a_k) - \alpha(Q(s_k, a_k) - y_k), \quad \forall k \in \text{minibatch}$$

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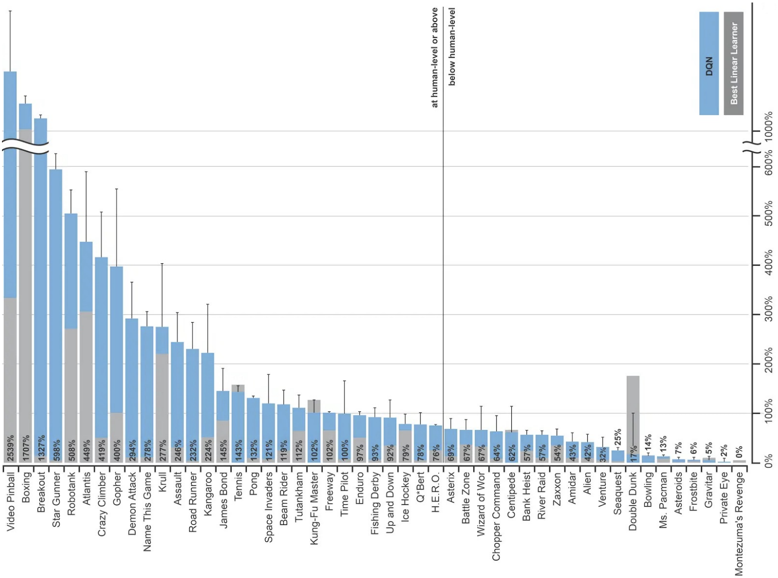
Results



DQN
Best Linear Learner
at human-level or above
below human-level
0%
100%
200%
300%
400%
500%
600%
1000%
Video Pinball 2539%
Boxing 1707%
Breakout 1327%
Star Gunner 598%
Robotank 508%
Atlantis 449%
Crazy Climber 419%
Gopher 400%
Demon Attack 294%
Name This Game 278%
Krull 277%
Assault 246%
Road Runner 232%
Kangaroo 224%
James Bond 145%
Tennis 143%
Pong 132%
Space Invaders 121%
Beam Rider 119%
Tutankham 112%
Kung-Fu Master 102%
Freeway 102%
Time Pilot 100%
Enduro 97%
Fishing Derby 93%
Up and Down 92%
Ice Hockey 79%
Q*Bert 78%
H.E.R.O. 76%
Asterix 69%
Battle Zone 67%
Wizard of Wor 67%
Chopper Command 64%
Centipede 62%
Bank Heist 57%
River Raid 57%
Zaxxon 54%
Amidar 43%
Alien 42%
Venture 32%
Seaquest 25%
Double Dunk 17%
Bowling 14%
Ms. Pacman 13%
Asteroids 7%
Frostbite 6%
Gravitar 5%
Private Eye 2%
Montezuma's Revenge 0%

Outline

Learning policies directly

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via stochastic gradient descent

Policy gradient theorem

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How do we efficiently compute/approximate it?

Policy gradient theorem (cont.)

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&= \sum_\tau P_\rho(\tau) \nabla_\rho \log P_\rho(\tau) R(\tau) && \text{(log derivative trick)} \\
&= \mathbb{E}_\tau \left[\nabla_\rho \log P_\rho(\tau) R(\tau) \right] && \text{(written as an } \textbf{expectation}\text{)}
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$$= \mathbb{E}_\tau \left[\nabla_\rho \log P_\rho(\tau) R(\tau)\right]$$ (written as an **expectation**)

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which can be approximated by sampling n trajectories using π_ρ and taking the empirical average:

$$\frac{1}{n} \sum_{i=1}^{n} \left(\sum_{h=1}^{H} \nabla_\rho \log \pi_\rho(a_h^{(i)} | s_h^{(i)}) \right) R(\tau^{(i)})$$

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This holds *for any b that only depends on $s_{1:h}, a_{1:h-1}$*, because

$$\mathbb{E}_{a_h} \left[\nabla_\rho \log \pi_\rho(a_h|s_h) b \right] = b \sum_{a_h \in \mathcal{A}} \pi_\rho(a_h|s_h) \frac{\nabla_\rho \pi_\rho(a_h|s_h)}{\pi_\rho(a_h|s_h)}$$

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Which baselines?

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Actor-Critic methods

Repeat:

- Critic evaluates the current policy π_ρ by fitting V_θ from samples using square loss:

$$\min_\theta \sum_{j=1}^{m} \sum_{h=1}^{H} \left(V_\theta\left(s_h^{(j)}\right) - \sum_{h'=h}^{H} r\left(s_{h'}^{(j)}, a_{h'}^{(j)}\right) \right)^2$$

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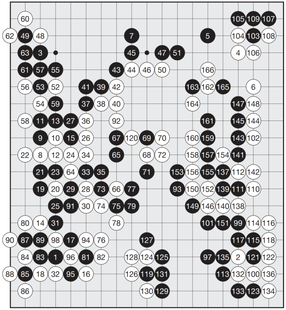
- Actor improves the current policy π_ρ via stochastic gradient descent:

$$\rho \leftarrow \rho - \frac{\alpha}{n} \sum_{i=1}^{n} \sum_{h=1}^{H} \nabla_\rho \log \pi_\rho(a_h^{(i)} | s_h^{(i)}) \underbrace{\left(\sum_{h'=h}^{H} r \left(s_{h'}^{(i)}, a_{h'}^{(i)} \right) - V_\theta(s_h^{(i)}) \right)}_{= R(\tau^{(i)}) - b(s_{1:h}^{(i)}, a_{1:h-1}^{(i)})}$$

Case study: AlphaGo

[Deepmind, 2015]

Model Go as an MDP $(\mathcal{S}, \mathcal{A}, P, r, \gamma)$:



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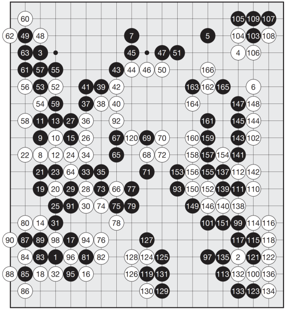

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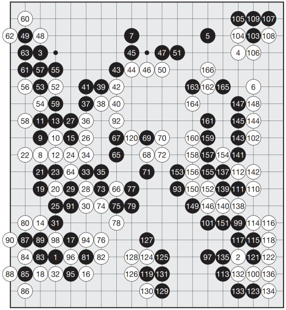

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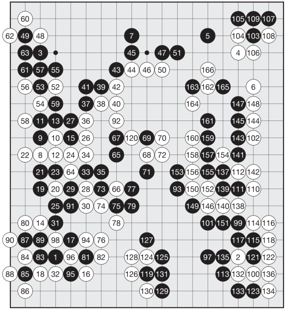

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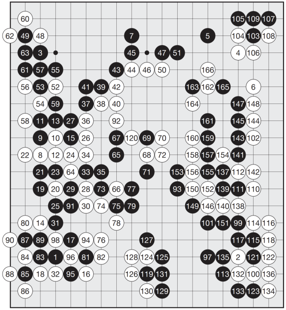
- reward: only the ending state has reward (1 if win, -1 if lose)



Case study: AlphaGo

[Deepmind, 2015]

Model Go as an MDP $(\mathcal{S}, \mathcal{A}, P, r, \gamma)$:

- states: each 19×19 position of the game is pre-processed into an $19 \times 19 \times 48$ image stack consisting of feature planes
- actions: all legal next moves
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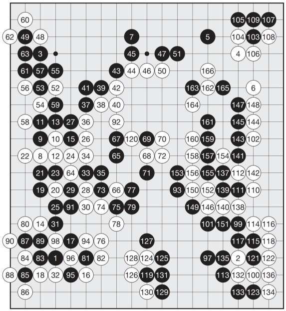
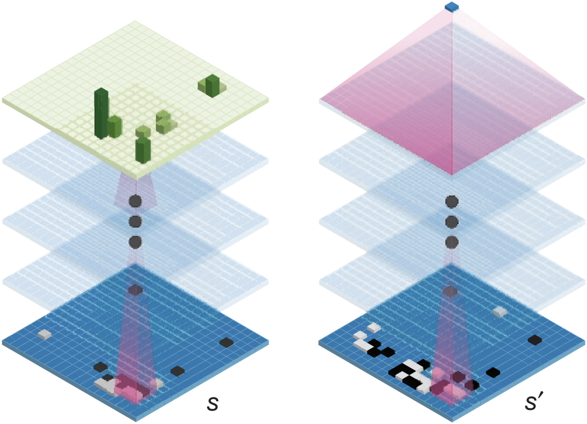
- $\gamma = 1$



Policy/value networks

Both π_ρ and V_θ are large convolutional neural nets:



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- trained for 10K iterations, each with 128 games

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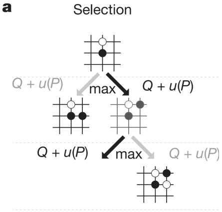

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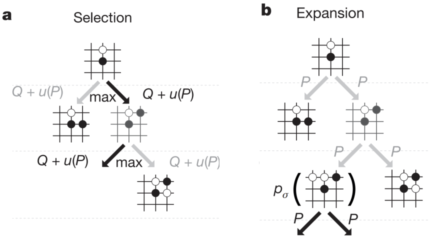

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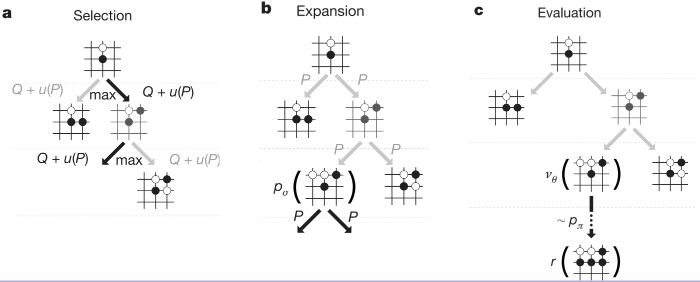

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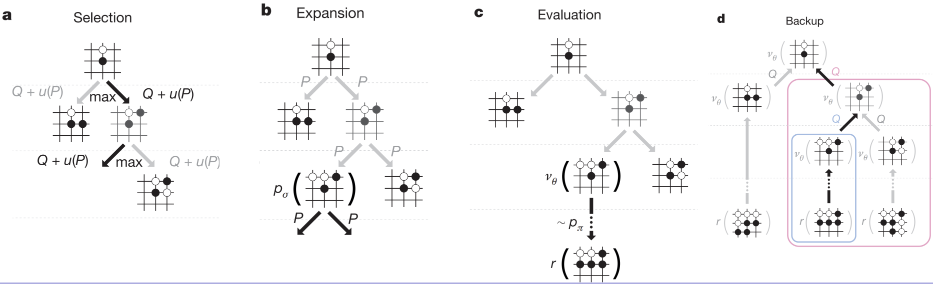

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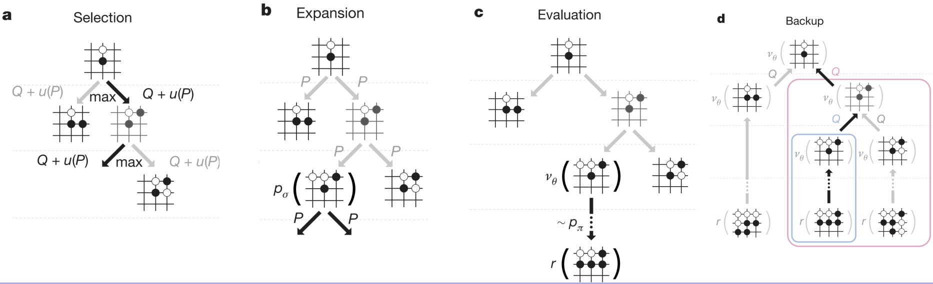

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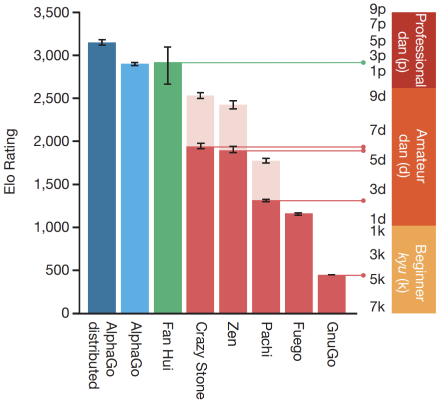

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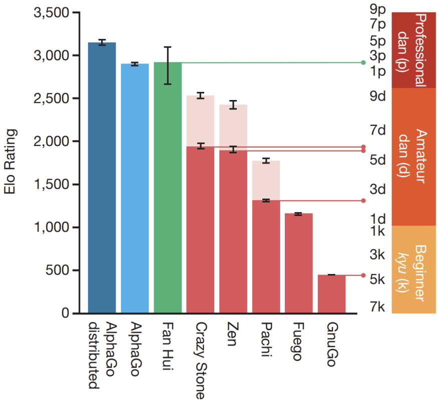

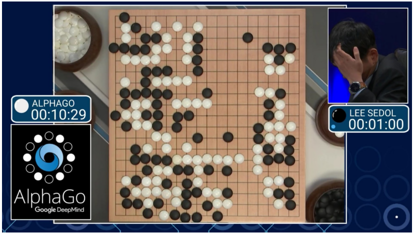

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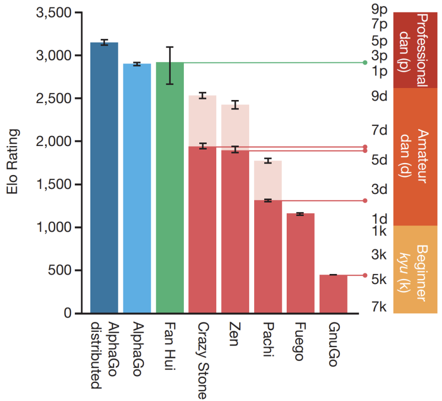
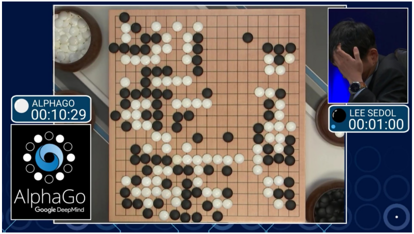
- first superhuman AI for Go, previously believed to be a decade away





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  - policy gradient, actor-critic methods, and their success in Go