Week 3 Practice

CSCI 567 Machine Learning

Spring 2025

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1. **MULTIPLE-CHOICE QUESTIONS: One or more correct choice(s) for each question.**

1.1. Which of the following surrogate losses is not an upper bound of the 0-1 loss?

(a) exponential loss: $\exp(-z)$

(b) hinge loss: $\max\{0, 1-z\}$

(c) perceptron loss: $\max\{0, -z\}$

(d) logistic loss: $\ln(1+\exp(-z))$

Ans: c, d. Note that here the logistic loss is using e as the base, instead of 2.

1.2. The perceptron algorithm makes an update $\boldsymbol{w}' \leftarrow \boldsymbol{w} + \eta y_n \boldsymbol{x}_n$ with $\eta = 1$ when $\boldsymbol{w}$ misclassifies $\boldsymbol{x}_n$. Using which of the following different values for η will make sure $\boldsymbol{w}'$ classifies $\boldsymbol{x}_n$ correctly?

(a) $\eta > \frac{y(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_n)}{\|\boldsymbol{x}_n\|_2^2}$ (b) $\eta < \frac{-y(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_n)}{\|\boldsymbol{x}_n\|_2^2+1}$

(c) $\eta < \frac{-y(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_n)}{\|\boldsymbol{x}_n\|_2^2}$ (d) $\eta > \frac{-y(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_n)}{\|\boldsymbol{x}_n\|_2^2}$

Ans: d. Solve $y_n \boldsymbol{w}'^{\mathrm{T}}\boldsymbol{x}_n = y_n(\boldsymbol{w} + \eta y_n \boldsymbol{x}_n)^{\mathrm{T}}\boldsymbol{x}_n > 0$ for η.

1.3. Which of the following is true?

(a) Normalizing the output $\boldsymbol{w}$ of the perceptron algorithm so that $\|\boldsymbol{w}\|_2 = 1$ changes its test error.

(b) Normalizing the output $\boldsymbol{w}$ of the perceptron algorithm so that $\|\boldsymbol{w}\|_1 = 1$ changes its

test error.
(c) When the data is linearly separable, logistic loss (without regularization) does not admit a minimizer.
(d) Minimizing 0-1 loss is generally NP-hard.

Ans: c, d. For c, note that when the data is separable, one can find $\boldsymbol{w}$ such that $y_n \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}_n \geq 0$ for all n. Scaling this $\boldsymbol{w}$ up will always lead to smaller logistic loss $\sum_{n=1} \ln(1 + \exp(-y_n \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}_n))$ and thus the function does not admit a minimizer.

1.4. Which of the following statement is correct for function $f(\boldsymbol{w}) = w_1 w_2$?
(a) $(0, 0)$ is the only stationary point.
(b) $(0, 0)$ is a local minimizer.
(c) $(0, 0)$ is a local maximizer.
(d) $(0, 0)$ is a saddle point.

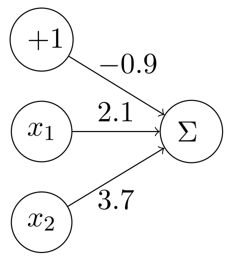
Ans: a, d. The gradient is $\nabla f(\boldsymbol{w}) = (w_2, w_1)$, so the only stationary point is $(0, 0)$. To see why it is neither a local minimizer nor a local maximizer, simply consider the direction $w_1 = -w_2$ and $w_1 = w_2$ respectively.

2. Perceptron

Consider the following training dataset:

x	y
(0, 0)	-1
(0, 1)	-1
(1, 0)	-1
(1, 1)	1

and a perceptron with weights $(w_0, w_1, w_2) = \{-0.9, 2.1, 3.7\}$



2.1. What is the accuracy of the perceptron on the training data?

SOLUTION:

x	y	$\hat{y}$	$\mathbb{I}(y = \hat{y})$
(0, 0)	-1	$sgn(-0.9) = -1$	Y
(0, 1)	-1	$sgn(-0.9 + 3.7) = 1$	N
(1, 0)	-1	$sgn(-0.9 + 2.1) = 1$	N
(1, 1)	1	$sgn(-0.9 + 2.1 + 3.7) = 1$	Y

Out of four predictions, two are correct. The accuracy is hence 50%.

2.2. Select $\mathbf{x} = (1, 0)$ and $y = -1$. Use the perceptron training rule with $\eta = 1$ to train the perceptron for one iteration. What are the weights after this iteration?

For the given $\mathbf{x} = (1, 0)$ the classifier makes a mistake ($\hat{y} = 1$). We need to update the weights following the perceptron rule the new weights are given by

$$\begin{aligned} w_{k+1} &\leftarrow w_k + \eta y \mathbf{x} \\ &\leftarrow (-0.9, 2.1, 3.7) + (1)(-1)(1, 1, 0) \\ &\leftarrow (-1.9, 1.1, 3.7) \end{aligned}$$

2.3. What is the accuracy of the perceptron on the training data after this iteration? Does the accuracy improve?

SOLUTION:

$\mathbf{x}$	y	$\hat{y}$	$\mathbb{I}(y = \hat{y})$
(0, 0)	-1	$sgn(-1.9) = -1$	Y
(0, 1)	-1	$sgn(-1.9 + 3.7) = 1$	N
(1, 0)	-1	$sgn(-1.9 + 1.1) = -1$	Y
(1, 1)	1	$sgn(-1.9 + 1.1 + 3.7) = 1$	Y

With the new weights, three out of four are correct, hence accuracy increased to 75%.

3. Maximum Likelihood Estimation

A random sample set $X_1, X_2, \ldots, X_n$ of size n is taken from a Poisson distribution with a mean of $\lambda > 0$. As a reminder, a Poisson distribution is a discrete probability distribution over the natural numbers, with the following probability mass function

$$P(X = x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}, \ \forall x \in \{0, 1, 2, \ldots, \}$$

3.1. Find the log likelihood of the data; call it $l(\lambda)$. You may use any log base you want.

$$\begin{aligned} l(\lambda) &= \log \prod_i P(X = X_i) \\ &= \log \prod_i \frac{\lambda^{X_i} e^{-\lambda}}{X_i!} \\ &= \sum_{i=1}^{n} [X_i \log \lambda - \lambda - \log(X_i!)] \\ &= \log \lambda \cdot \sum_{i=1}^{n} X_i - n\lambda - \sum_{i=1}^{n} \log(X_i!) \end{aligned}$$

3.2. Find the maximum likelihood estimator for λ.

$$\begin{aligned} l'(\lambda) &= \frac{1}{\lambda} \sum_{i=1}^{n} X_i - n = 0 \\ \lambda &= \frac{1}{n} \sum_{i=1}^{n} X_i \end{aligned}$$

So our maximum likelihood estimator $\hat{\lambda}$ is the average value.